Written HW12 – MATH 2501 Fall 2020

Due by 20 October for timely completion credit

Each of these problems has you finding minima or maxima of a described situation. Full credit is only awarded when they are solved entirely symbolically. If a calculator is used, then the output of the calculator must be included. In each problem, an appropriate test (first or second derivative test or extreme value theorem) must be used to prove the value is an extremum.

1. A piece of wire $15m$ long is cut into two pieces. One piece is bent into a square and the other is bent into an equilateral triangle. How should the wire be cut so that the total area enclosed by the two shapes is a maximum? a minimum?

2. In a beehive, each cell is a regular hexagonal prism, open at one end with a trihedral angle at the other end. It is believed that bees form their cells in such a way as to minimize the surface area for a given volume, thus using the least amount of wax in cell construction. Examination of these cells has shown that the measure of the apex angle θ is amazingly consistent. Based on the geometry of the cell, it can be shown that the surface area S is given by

$$S = 6sh - \frac{3}{2}s^2\cot(\theta) + \left(\frac{3\sqrt{3}s^2}{2}\right)\csc(\theta),$$

where s (side lengths of the hexagons) and h (height of cell) are constants. Determine the minimum surface area of the cell (as a function of s and h). (*note: in actual measurements, the angle θ found here in beehives rarely differ from the calculated value by more than* 2°)