Ex: $F(x) = \int_{x-1}^{\sin(\pi x)} t^2\,dt$

$$\int_a^b = \int_a^c + \int_c^b$$

$$-\int_a^b = \int_b^a$$

Compute $\frac{d}{dx}F(x)$.

Soln: Calculate

need to match

$$\frac{d}{dx}F(x) = \frac{d}{dx}\int_{x-1}^{\sin(\pi x)} t^2\,dt$$

needs to be a number

$$= \frac{d}{dx}\left[\int_{x-1}^{0} t^2\,dt + \int_{0}^{\sin(\pi x)} t^2\,dt\right]$$

$$= \frac{d}{dx}\left[-\int_{0}^{x-1} t^2\,dt + \int_{0}^{\sin(\pi x)} t^2\,dt\right]$$

$$= -\frac{d}{dx}\int_{0}^{x-1} t^2\,dt + \frac{d}{dx}\int_{0}^{\sin(\pi x)} t^2\,dt$$

$$= -\left(\frac{d(x-1)}{dx}\right)\frac{d}{d(x-1)}\int_{0}^{x-1} t^2\,dt + \frac{d(\sin(\pi x))}{dx}\,\frac{d}{d(\sin(\pi x))}\int_{0}^{\sin(\pi x)} t^2\,dt$$

$$= -1\left((x-1)^2\right) + \pi\cos(\pi x)\left(\sin(\pi x)\right)^2$$

Ex: Calculate

$$\int_a^a f(t)\,dt = 0$$

$$\frac{d}{dx}\int_x^x \sin(t)\,dt$$

short way $\rightarrow$ $\left(= \frac{d}{dx}(0) = 0 \right)$

Soln (long way)

$$\frac{d}{dx}\int_x^x \sin(t)\,dt = \frac{d}{dx}\left[\int_x^0 \sin t\,dt + \int_0^x \sin t\,dt\right]$$

$$= -\frac{d}{dx}\int_0^x \sin(t)\,dt + \frac{d}{dx}\int_0^x \sin t\,dt$$

$$= -\sin(x) + \sin(x)$$

$$= 0$$

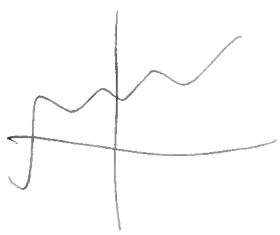
Ex: $\int_0^2 t^2+3t+1\,dt$

Soln: Using FTC

plug in 2nd

$$\int_0^2 t^2+3t+1\,dt = \underbrace{D^{-1}(t^2+3t+1)}_{\text{1st you anti-diff}}\Big|_0^2$$

subtract plugging this in 3rd



$$= \frac{t^3}{3}+\frac{3t^2}{2}+t+C\Big|_0^2$$

this will actually always cancel in all such problems

$$= \left(\frac{2^3}{3}+\frac{3(2^2)}{2}+2+\cancel{C}\right)$$

$$-\left(\cancel{\frac{0^3}{3}+\frac{3(0^2)}{2}+0}^{\,0}+\cancel{C}\right)$$

$$\begin{array}{r} 28 \\ 36 \\ \hline 64 \end{array}$$

$$= \frac{8}{3}+\frac{12}{2}+2$$

$$= \frac{16}{6}+\frac{36}{6}+\frac{12}{6} = \frac{64}{6} = \frac{32}{3}$$

Ex: $\displaystyle\int_{-2}^{5} e^{t}\,dt = D^{-1}(e^{t})\Big|_{-2}^{5}$

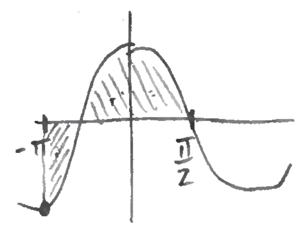
$= e^{t}+C\Big|_{-2}^{5}$

$= (e^{5}+\cancel{C}) - (e^{-2}+\cancel{C})$

$= e^{5}-e^{-2}$

Ex: $\displaystyle\int_{-\pi}^{\pi/2} \cos(t)\,dt = D^{-1}(\cos(t))\Big|_{-\pi}^{\pi/2}$

$= \sin(t)\Big|_{-\pi}^{\pi/2}$



$= \sin\left(\frac{\pi}{2}\right) - \sin(-\pi)$

$= \underline{1} - 0$

$= \underline{1}$

5

Ex: $\int_1^2 \frac{d}{dt} e^{\sin(t)}\, dt$

$= \cancel{D^{-1}}\left(\cancel{\frac{d}{dt}} e^{\sin t}\right)\Big|_1^2$

$= e^{\sin(t)}\Big|_1^2$

$= e^{\sin(2)} - e^{\sin(1)}$

Ex: Compute $\lim\limits_{x\to 0^+} \dfrac{\int_0^x t^2 \sin(t)\, dt}{x^3}$ $\xrightarrow{\text{set } x=0}$ $\dfrac{\int_0^0 \sim dt}{0^3} = \dfrac{0}{0}$

more work to do

L.H. $= \lim\limits_{x\to 0^+} \dfrac{\frac{d}{dx}\int_0^x t^2 \sin(t)\, dt}{\frac{d}{dx}(x^3)}$

$= \lim\limits_{x\to 0^+} \dfrac{\cancel{x^2}\sin(x)}{3\cancel{x^2}} = \frac{1}{3}\lim\limits_{x\to 0^+} \sin(x) = 0$

Fractional calculus

IDEA: What does

$$\frac{d^{1/2}}{dx^{1/2}} f(x)$$

mean?

$$\frac{d^2}{dx^2} f(x)$$

$$\frac{d^1}{dx^1}\frac{d^1}{dx^1} f(x) = \frac{d^2}{dx^2} f(x)$$

$$\frac{d^{1/2}}{dx^{1/2}}\frac{d^{1/2}}{dx^{1/2}} f(x) = \frac{d}{dx} f(x)$$

$$\frac{d^2}{dx^2} x^3 = 3 \cdot 2\, x^{3-2}$$

Polynomials

$$\frac{d}{dx} x^1 = 1x^{1-1}$$

$$\frac{d^{1/2}}{dx^{1/2}} x^1 = (\text{const})\, x^{1-1/2} = (\text{const})\sqrt{x}$$

$$\frac{d^{1/2}}{dx^{1/2}} x^{1/2} = (\text{const})\, x^{1/2-1/2} = \underbrace{(\text{const})}_{=1?}$$

Gamma function

IDEA: can we extend a factorial to non-integer inputs?

$3! = 3 \cdot 2 \cdot 1$

$4! = 4 \cdot 3 \cdot 2 \cdot 1$

$n \geq 1$ integer

$n! = n(n-1)(n-2) \cdots 3 \cdot 2 \cdot 1$