**Ex**: Find a formula for a function $f(x)$ such that $f'(x) = \frac{\sin(x)}{x}$ and $f(3) = 7$.

**Soln**: Use FTC to realize that the function $f(x)$ can take form

$$f(x) = \int_3^x \frac{\sin(t)}{t}\,dt + C$$

(works b/c

$$\frac{d}{dx} f(x) = \frac{d}{dx}\left[\int_3^x \frac{\sin(t)}{t}\,dt + \overset{0}{\cancel{C}}\right] \overset{FTC}{=} \frac{\sin(x)}{x}$$

)

Now consider finding the value of C.

$$\underset{\text{given}}{7} = f(3) \underset{\text{compute}}{=} \underbrace{\int_3^3 \frac{\sin(t)}{t}\,dt}_{=0} + C$$

$\Rightarrow 7 = C$. So

$$\boxed{f(x) = 7 + \int_3^x \frac{\sin(t)}{t}\,dt}$$

$$\int_a^c = \int_a^b + \int_b^c$$

$$-\int_a^b = \int_b^a$$

if instead you used

$$f(x) = \int_{-2}^x \frac{\sin(t)}{t}\,dt + C$$

$$\underset{\text{given}}{7} = f(3) = \underbrace{\int_{-2}^3 \frac{\sin(t)}{t}\,dt}_{\text{a number}} + C$$

corr. to area

$$\Rightarrow C = 7 - \int_{-2}^3 \frac{\sin(t)}{t}\,dt$$

$$\Rightarrow f(x) = 7 - \int_{-2}^3 \frac{\sin(t)}{t}\,dt + \int_{-2}^x \frac{\sin(t)}{t}\,dt$$

$$= 7 + \int_3^{-2} \frac{\sin(t)}{t}\,dt + \int_{-2}^x \frac{\sin(t)}{t}\,dt$$

Ex: Let

$$f(t)=\int_0^t \frac{x^2+3x+2}{1+\cos^2(x)}\,dx$$

Find local extrema.

Soln: $\frac{d}{dt}f(t)=\frac{d}{dt}\int_0^{t}\frac{x^2+3x+2}{1+\cos^2(x)}\,dx$

match

$$=\boxed{\frac{t^2+3t+2}{1+\cos^2(t)} \overset{\text{set}}{=} 0}$$

$1+\cos^2(t)\neq 0$

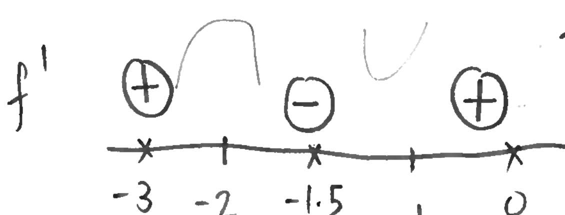
mult by $1+\cos^2(t)$

$$t^2+3t+2=0$$

$$(t+2)(t+1)=0$$

$$t=-2,\ t=-1$$

Use 1st deriv test:




$\Rightarrow$ f has

loc max at $t=-2$

loc min at $t=-1$

Ex: $f(x) = \int_0^x e^t + t^2 \, dt$

What is $f''(x)$?

Soln:

$$f'(x) = \frac{d}{dx} \int_0^{x} e^t + t^2 \, dt$$

match!

$$= e^x + x^2$$

$$f''(x) = \frac{d}{dx}\left[e^x + x^2\right] = e^x + 2x$$

Ex: Compute $\int_0^x e^t + t^2 \, dt$ "$f'(t)$"

Soln: $D^{-1}(e^t + t^2) = e^t + \frac{t^3}{3} + C$ "$f(t)$"

By (1) FTC:

$$\int_0^{x} e^t + t^2 \, dt = \underbrace{\left(e^x + \frac{x^3}{3} + \cancel{C}\right)}_{f(x)} - \underbrace{\left(e^0 + \frac{0^3}{3} + \cancel{C}\right)}_{f(0)}$$

$$= e^x + \frac{x^3}{3} - 1$$

Ex: Let $f(x) = \int_0^{x^2} t\sin(t)\,dt$

Compute $f'(x)$. → chain rule

mismatch!!

Soln: $f'(x) = \frac{d}{dx}\int_0^{x^2} t\sin(t)\,dt$

match!

$= \frac{d(x^2)}{dx}\,\frac{d}{dx}\int_0^{x} t\sin(t)\,dt$

$= 2x(x\sin(x))$

$= 2x^3\sin(x)$

next time:

$\frac{d}{dx}\int_{F(x)}^{G(x)} \sim dt$