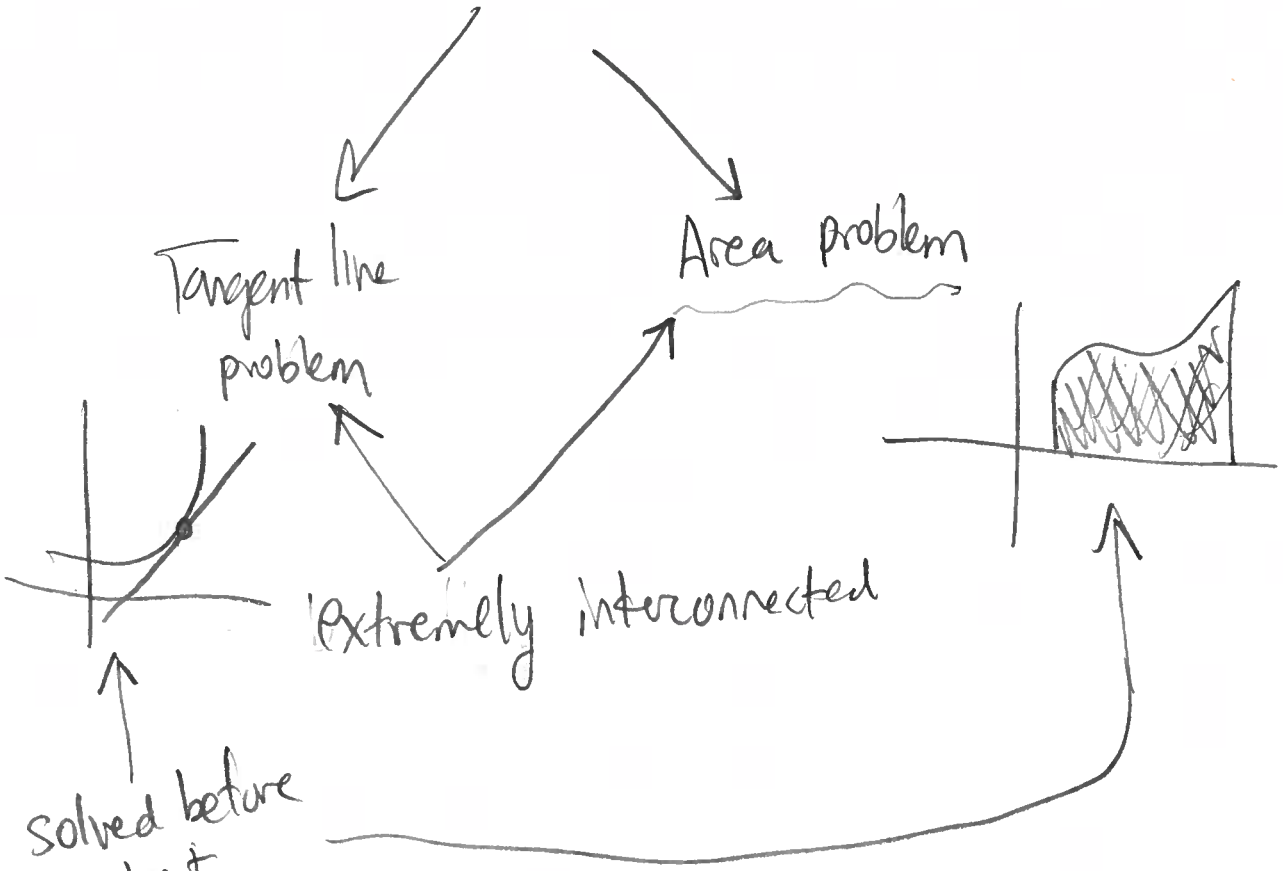
What is calculus?



so what did they do?

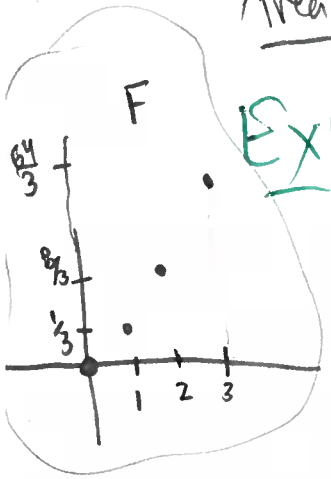
Fundamental Theorem of Calculus

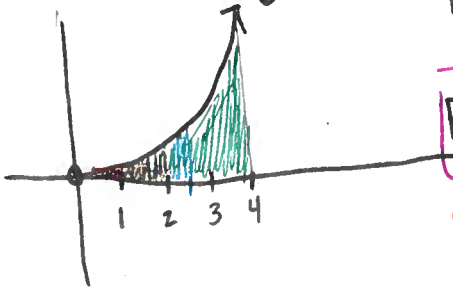
$$\sum_{K=1}^{2} a_K = a_1 + a_2$$

Area functions

t is a "dummy variable"

Ex: $F(x) = \int_0^x t^2\,dt$





$F(0) = \int_0^0 t^2\,dt = 0$

$F(1) = \int_0^1 t^2\,dt = \frac{1}{3}$

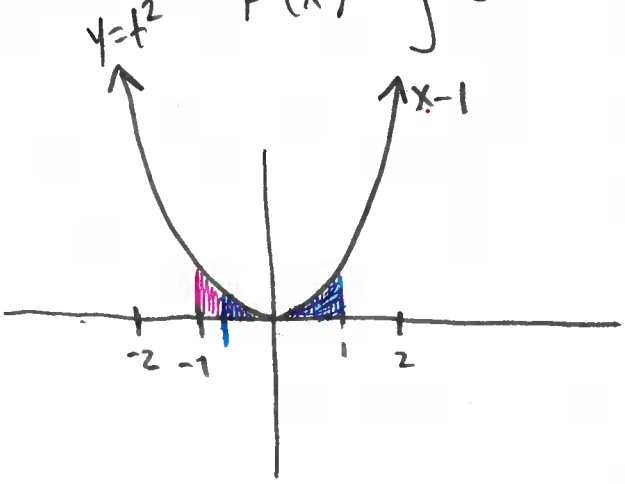
$F(2) = \int_0^2 t^2\,dt = \frac{8}{3}$

$F(2.5) = \int_0^{2.5} t^2\,dt = \frac{(2.5)^3}{3}$

$F(4) = \int_0^4 t^2\,dt = \frac{64}{3}$

Ex: "Weird" area fnct

$$F(x) = \int_{-1}^{\sin(\pi x)} t^2\,dt$$



$$F(0) = \int_{-1}^{\sin(0)} t^2\,dt = \int_{-1}^{0} t^2\,dt = \frac{1}{3}$$

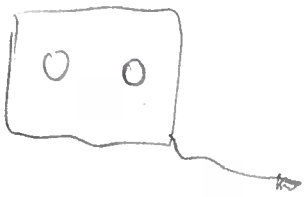
$$F\left(\tfrac{1}{2}\right) = \int_{-1}^{\sin(\frac{\pi}{2})} t^2\,dt = \int_{-1}^{1} t^2\,dt$$

$$F(2) = \int_{1}^{\sin(2\pi)} t^2\,dt = \int_{1}^{0} t^2\,dt$$

$$= -\int_{0}^{1} t^2\,dt$$

Many commonly used functions have a representation as an area function!

Fresnel function: $S(x) = \int_0^x \sin(t^2)\,dt$ (optics)



Sine integral: $Si(x) = \int_0^x \frac{\sin(t)}{t}\,dt$ (low pass filter)

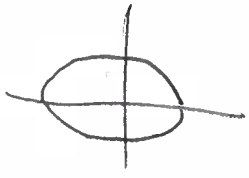
$$\operatorname{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2}\,dt$$

(error function)

Elliptic integral of 2nd kind

$$E(x) = \int_0^x \frac{\sqrt{1-k^2t^2}}{\sqrt{1-t^2}}\,dt$$

↑ circumference of an ellipse



Fundamental theorem of calculus

Shows us how derivatives & integrals interact.

Two flavors:

$f = D^{-1} f'$

"integral of derivative" (1) $\int_a^b f'(t)\,dt = f(b) - f(a)$

match

"derivative of integral" (2) $\frac{d}{dx}\int_a^x f(t)\,dt = f(x)$