Properties of integrals $\int_a^b f(x)\,dx$

1

Recall:

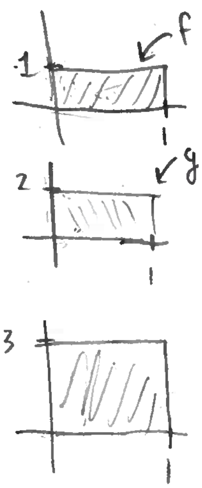
$$\frac{d}{dx}[f+g] = \frac{df}{dx} + \frac{dg}{dx}$$

$$\frac{d}{dx}[\alpha f] = \alpha \frac{df}{dx}$$

constant

linear property of $\frac{d}{dx}$

We have similar properties for integrals



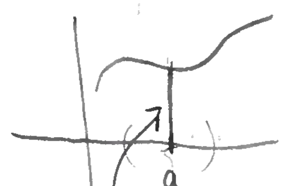
$$\int_a^b f(x)+g(x)\,dx = \int_a^b f(x)\,dx + \int_a^b g(x)\,dx$$

$$\int_a^b \alpha f(x)\,dx = \alpha \int_a^b f(x)\,dx$$

constant

linear property of $\int$

$$\int_a^a f(x)\,dx = 0$$



rectangle w/ height $f(a)$ and infinitesimally small width

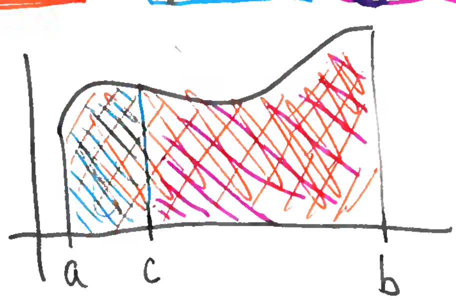
is there a version of product rule for $\int$? can I compute $\int fg\,dx$? yes

is there a $\int$ version of chain rule? yes

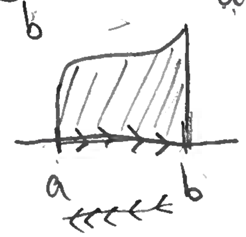
Calc 2

c ← match

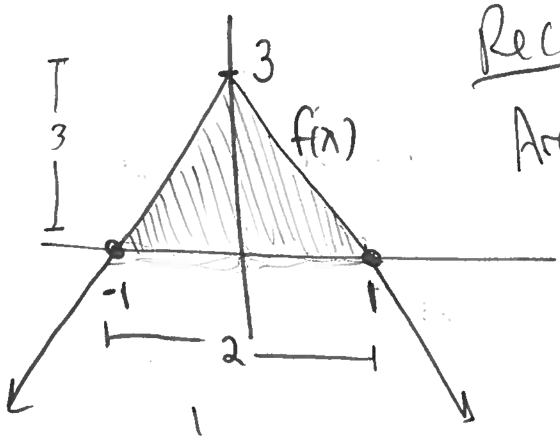
$$\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx$$



$$\int_b^a f(x)\,dx = -\int_a^b f(x)\,dx$$

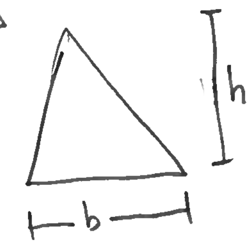


Ex: Consider the function



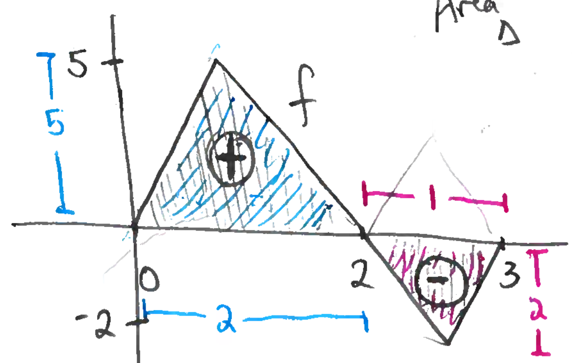
Recall

$\text{Area}_\Delta = \frac{1}{2}bh$



Compute $\int_{-1}^{1} f(x)\,dx \underset{\text{Area}_\Delta}{=} \frac{1}{2}(2)(3) = 3$

Ex:



Compute $\int_{0}^{3} f(x)\,dx$.

Soln: Compute

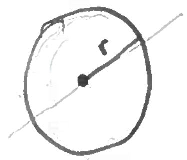
$$\int_0^3 f(x)\,dx = \int_0^2 f(x)\,dx + \int_2^3 f(x)\,dx$$

$$= \frac{1}{2}(2)(5) - \frac{1}{2}(1)(2)$$

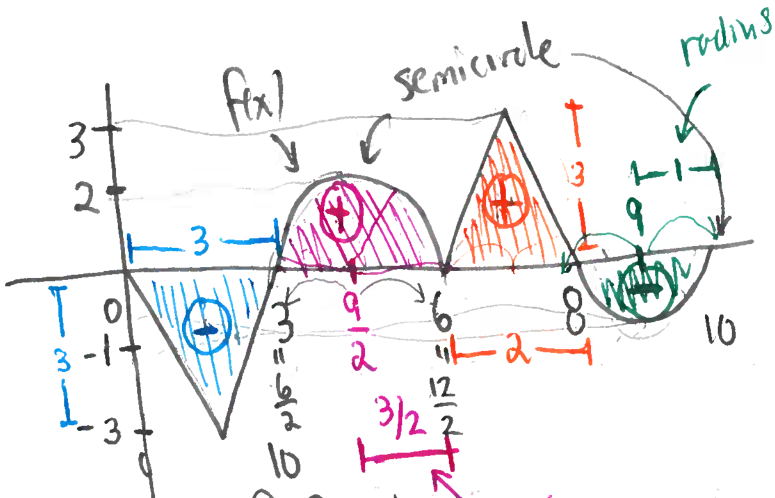
$$= 5 - 1 = 4$$

Ex:

$\text{Area}_{\bigcirc} = \pi r^2$



$\text{Area}_{\frown} = \frac{1}{2}\pi r^2$



radius!

midpoint of 3 and 6

$\frac{3+6}{2} = \frac{9}{2}$

Compute $\int_0^{10} f(x)\,dx$

Soln:

$$\int_0^{10} f(x)\,dx = \int_0^3 f(x)\,dx + \int_3^{10} f(x)\,dx$$

$$= \int_0^3 f(x)\,dx + \int_3^6 f(x)\,dx + \int_6^{10} f(x)\,dx$$

$$= \int_0^3 f(x)\,dx + \int_3^6 f(x)\,dx + \int_6^8 f(x)\,dx + \int_8^{10} f(x)\,dx$$

$$= -\frac{1}{2}(3)(3) + \frac{1}{2}\pi\left(\frac{3}{2}\right)^2 + \frac{1}{2}(2)(3) - \frac{1}{2}\pi(1^2)$$

$$= -\frac{9}{2} + \frac{9}{8}\pi + 3 - \frac{\pi}{2}$$

What's next?

Fundamental theorems of calculus

↓

allow us to answer questions like

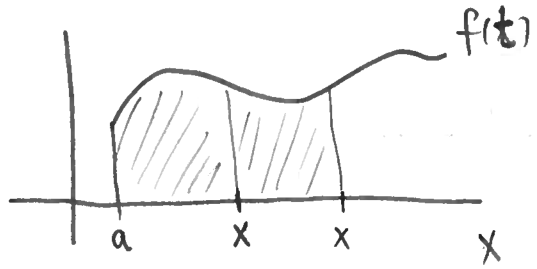
$\int_0^5 x^2+1\,dx$ w/o limits ✓

and

$\frac{d}{dx}\int_1^x f(t)\,dt$

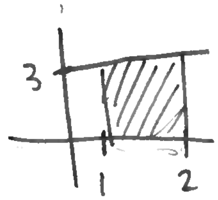
What is going on with $\int_1^x f(t)\,dt$

called an area function

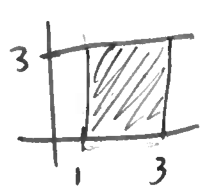


Could define $F(x)=\int_1^x 3\,dt$

$F(2)=3$



$F(3)=6$



$F(4)=9$