Antiderivatives of polynomials

From last time:

$$D^{-1}x^n = \frac{x^{n+1}}{n+1} + C \; ; \; n \neq -1$$

(note: if $n=-1$, then

$$D^{-1}x^{-1} = D^{-1}\left(\frac{1}{x}\right) = \ln(|x|)$$

may need abs value for webwork)

FACTS

* $D^{-1}(f+g) = (D^{-1}f) + (D^{-1}g)$
* $D^{-1}(\alpha f) = \alpha(D^{-1}f)$

α constant

(antidiff is a "linear transformation")

Ex: Find antideriv of x^2+3x+1

Soln:

$1 = x^0$

$$D^{-1}(x^2+3x+1) = D^{-1}(x^2) + D^{-1}(3x) + D^{-1}(1)$$

$$= D^{-1}(x^2) + 3D^{-1}(x) + D^{-1}(1)$$

$$= \left(\frac{x^3}{3} + C\right) + 3\left(\frac{x^2}{2} + D\right) + \left(\frac{x^1}{1} + E\right)$$

$$= \frac{x^3}{3} + \frac{3x^2}{2} + x + \tilde{C} \; ; \text{ where } \tilde{C} = C + 3D + E$$

Ex: Find antideriv of $x^7+3x^4-29x^3+x-14$

Soln: Compute

$$D^{-1}\left(x^7+3x^4-29x^3+x-14\right)$$

linear property of D^{-1}

$$= D^{-1}(x^7)+3D^{-1}(x^4)-29D^{-1}(x^3)+D^{-1}(x)-14D^{-1}(1)$$

$$= \frac{x^8}{8}+\frac{3x^5}{5}-\frac{29x^4}{4}+\frac{x^2}{2}-14x+C$$

Exponentials and logs:

$$\boxed{\frac{d}{dx}e^x=e^x}$$

$$\boxed{D^{-1}e^x=e^x+C}$$

$$\frac{d}{dx}\ln(x)=\frac{1}{x}$$

$$D^{-1}\left(\frac{1}{x}\right)=\ln(|x|)+C$$

Ex:

$$\frac{d}{dx}e^{ax}=\frac{d(ax)}{dx}\frac{d}{d(ax)}e^{ax}$$

$$= ae^{ax}$$

$$D^{-1}(ae^{ax})=e^{ax}+C$$

$$\boxed{D^{-1}(e^{ax})=\frac{e^{ax}}{a}+\tilde{C}\ ;\ \tilde{C}=\frac{C}{a}}$$

trouble
$D^{-1}(xe^{x})$

Ex: Compute

$$D^{-1}\left(3e^{2x}+2e^{-5x}+\frac{1}{3x}+\frac{1}{6x}\right)$$

linearity

$$=3D^{-1}(e^{2x})+2D^{-1}(e^{-5x})+\frac{1}{3}D^{-1}\left(\frac{1}{x}\right)+\frac{1}{6}D^{-1}\left(\frac{1}{x}\right)$$

$$=\frac{3}{2}e^{2x}+\left(\frac{2}{-5}\right)e^{-5x}+\frac{1}{3}\ln(x)+\frac{1}{6}\ln(x)+C$$

$$=\frac{3}{2}e^{2x}-\frac{2}{5}e^{-5x}+\frac{1}{2}\ln(x)+C$$

Trig functions

turns out
$D^{-1}\tan x$
$\parallel$
$\ln(\cos(x))$

$\frac{d}{dx}\ln(-\cos(x))$
$\parallel$
$\frac{1}{\cos(x)}(+\sin(x))$

$$\frac{d}{dx}\sin x=\cos x$$

$$D^{-1}\cos(x)=\sin(x)+C$$

$$\frac{d}{dx}\cos(x)=-\sin(x)$$

$$D^{-1}(-\sin(x))=\cos(x)+C$$

$$D^{-1}(\sin(x))=-\cos(x)+\tilde{C};\ \tilde{C}=-C$$

Ex: Compute

$$D^{-1}(3\cos(x)+2\sin(x))$$

$$=3D^{-1}(\cos(x))+2D^{-1}(\sin(x))$$

$$=3\sin(x)-2\cos(x)+C$$

Quick intro to differential equations

operations: $+, -, \div, *$

Arithmetic $\longrightarrow$ Calculus

operations: $\frac{d}{dx}, D^{-1}$

Algebra $\longrightarrow$ Differential Eqt

relate unknowns by
operations of arithmetic
goal: find unknown

$x + 2 = 3$

number

relates unknowns by
operations of calculus
goal: find unknown

$$\boxed{\frac{d}{dt} x = t^2 + 1 \longrightarrow x = D^{-1}(t^2+1) \quad \text{soln: } x = \frac{t^3}{3} + t + C}$$

function

$x^2 = 9$

$x = \pm 3$

$x + 2 = 2x - 5$

$\frac{d}{dt} y = y \longrightarrow$ soln $y = Ce^t$

$-\sin t \xrightarrow{\frac{d}{dt}} \cos t \xrightarrow{\frac{d}{dt}} -\sin(t)$

$\frac{d^2}{dt^2} \sin t = -\sin(t)$

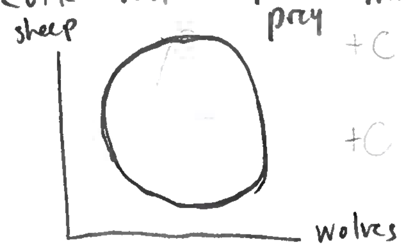
$\frac{d^2}{dt^2} y = -y \longrightarrow$ soln $y = c_1 \cos(t) + c_2 \sin(t)$

$$\begin{cases} x + y = 2 \\ x - y = 3 \end{cases}$$

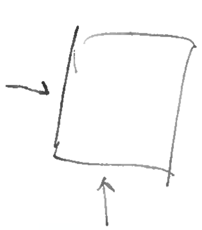
$\frac{d^2}{dt^2} y = ty \longrightarrow$ soln $y = c_1 Ai(t) + c_2 Bi(t)$

Airy functions

Lotke-Volterra predator prey model



$$\begin{cases} \frac{dx}{dt} + \frac{dy}{dt} = 8 \\ 3\frac{dx}{dt} - 2\frac{dy}{dt} = t \end{cases}$$



Ex: Find the antiderivative F of $f(x) = x^2 + 3x + 2$
such that $\boxed{F(0) = 3}$.

"initial condition"

Soln:

$$F(x) = D^{-1}(f(x))$$

$$= \frac{x^3}{3} + \frac{3x^2}{2} + 2x + C$$

given

computed

$$3 = F(0) = \frac{0^3}{3} + \frac{3(0^2)}{2} + 2(0) + C$$

$$3 = F(0) = C$$

$$\Rightarrow \boxed{C = 3}$$

$$\boxed{F(x) = \frac{x^3}{3} + \frac{3x^2}{2} + 2x + 3}$$