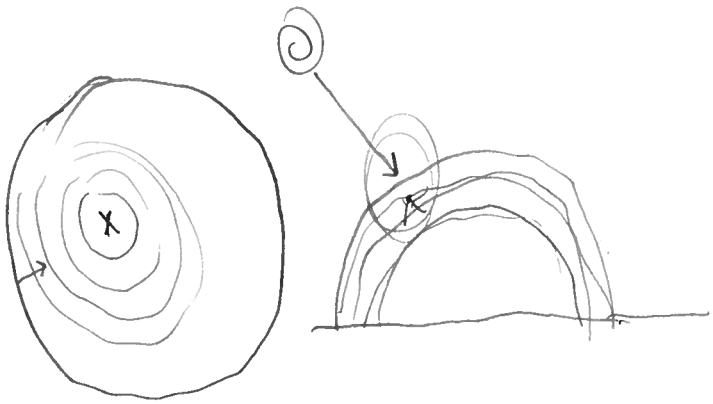


EX: The Airy Bi function has asymptotic equation

$$Bi(x) \approx \frac{e^{(2/3)x^{3/2}}}{\sqrt{\pi}\, x^{1/4}} \quad (\text{as } x \to \infty)$$

Calculate

$$\lim_{x\to\infty} e^{-x^2} Bi(x)$$

$e^a e^b = e^{a+b}$

Soln: Use asymptotic formula to write

$$\lim_{x\to\infty} e^{-x^2} Bi(x) = \lim_{x\to\infty} e^{-x^2}\left(\frac{e^{\frac{2}{3}x^{3/2}}}{\sqrt{\pi}\, x^{1/4}}\right)$$

$$= \frac{1}{\sqrt{\pi}} \lim_{x\to\infty} \frac{e^{-x^2+\frac{2}{3}x^{3/2}}}{x^{1/4}}$$

$\lim e^{p(x)} = e^{\lim p(x)}$

$$-x^2 + \frac{2}{3}x^{3/2} = x^{3/2}\left(\frac{2}{3} - x^{1/2}\right)$$

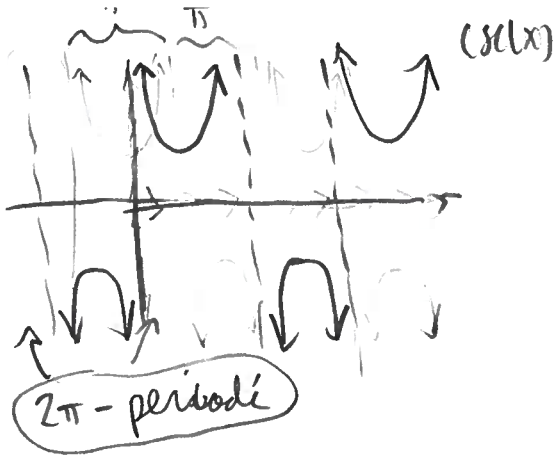
$$= \frac{1}{\sqrt{\pi}} \lim_{x\to\infty} \frac{e^{\frac{2}{3}x^{3/2}}}{e^{x^2} x^{1/4}} = 0 \qquad \frac{\infty}{\infty}$$

"Who wins race?"

$\left(e^{\frac{2}{3}x^{3/2}} \text{ or } e^{x^2}?\right)$

↑ winner – higher power on x

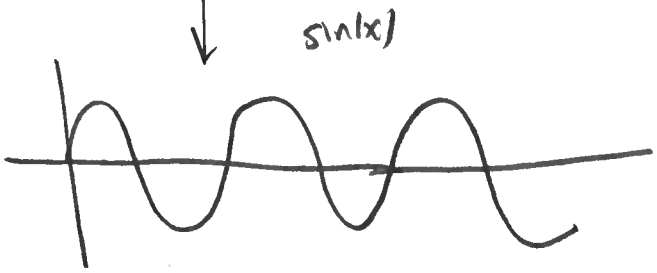
Ex: $\lim_{x\to\infty} e^{\csc(x)}$



DNE —
function repeats itself forever!

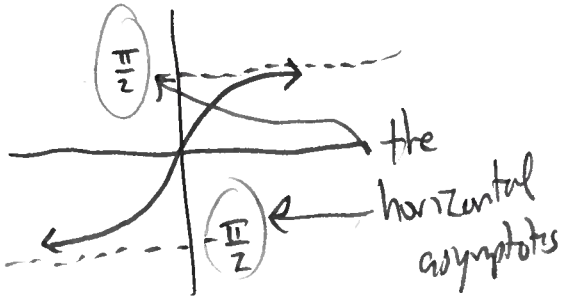
Ex: Similarly,

$\lim_{x\to\infty} \sin(x)$



DNE
— repeats itself forever!

"horizontal asymptote"
└ used sometimes when a function has a $\lim_{t\to\infty}$ or $\lim_{t\to-\infty}$ or BOTH



Ex: Show there is horiz. asympt by calculating...

$$\lim_{x\to\infty} \frac{3x+2}{x-9} + \frac{10x^2+3}{-5x^2-4} \overset{LH}{=} \lim_{x\to\infty}\left(\frac{3x+2}{x-9}\right) + \lim_{x\to\infty}\left(\frac{10x^2+3}{-5x^2-4}\right)$$

$\frac{\infty}{\infty}$ $\frac{\infty}{\infty}$ $\frac{\infty}{\infty}$ $\frac{\infty}{\infty}$

$$\overset{LH}{=} \lim_{x\to\infty}\left(\frac{3}{1}\right) + \lim_{x\to\infty}\left(\frac{20x}{-10x}\right)$$

$$= 3 - \frac{20}{10} = 3 - 2 = 1$$

Ex: $\lim_{x\to-\infty} \sqrt{x^2+3x-1} + x$ $\quad \infty - \infty$

$$\overset{\text{(Alg trick)}}{=} \lim_{x\to-\infty} \left(\sqrt{x^2+3x-1}+x\right)\left(\frac{\sqrt{x^2+3x-1}-x}{\sqrt{x^2+3x-1}-x}\right)$$

How to do L'Hôpital w/ no numerator+denom?

$$= \lim_{x\to-\infty} \frac{\left(\sqrt{x^2+3x-1}\right)^2 - x^2}{\sqrt{x^2+3x-1} - x}$$

$\frac{\infty-\infty}{\infty+\infty}$

$\left(\sqrt{x^2+3x-1}\right)^2 - x^2 \to x^2+3x-1-x^2 = 3x-1$

$$\frac{d}{dx}\left(x^2+3x-1\right)^{1/2}$$

$$= \frac{d\left(x^2+3x-1\right)}{dx}\,\frac{d}{d\left(x^2+3x-1\right)}\left(x^2+3x-1\right)^{1/2}$$

$$= (2x+3)\left(\tfrac{1}{2}\right)\frac{1}{\sqrt{x^2+3x-1}}$$

$$\overset{LH}{=} \lim_{x\to-\infty} \frac{3}{\frac{2x+3}{2\sqrt{x^2+3x-1}} - 1}$$

$$= \frac{3}{\frac{1}{2}\lim_{x\to-\infty}\frac{2x+3}{\sqrt{x^2+3x-1}} - 1} \qquad (\star)$$

(4)

Let's calculate $\lim_{x\to-\infty} \frac{2x+3}{\sqrt{x^2+3x-1}}$ (negative ~ "$-x$" is pos.)

$$= \lim_{x\to-\infty} \frac{2x+3}{\sqrt{x^2+3x-1}} \left(\frac{-\frac{1}{x}}{-\frac{1}{x}}\right)$$

$x<0$

$-\frac{1}{x}>0$

if $x<0$

$\sqrt{x^2} \neq x$

$\sqrt{x^2} = -x$

↑ always pos

$$= \lim_{x\to-\infty} \frac{-2-\frac{3}{x}}{\sqrt{\frac{x^2+3x-1}{x^2}}}$$

$$= \lim_{x\to-\infty} \frac{-2-\frac{3}{x}}{\sqrt{1+\frac{3}{x}-\frac{1}{x^2}}}$$

$= -2$

By (★),

$$\lim_{x\to-\infty} \sqrt{x^2+3x-1}+x = \frac{3}{\frac{1}{2}\left[\lim_{x\to\infty}\left(\frac{2x+3}{\sqrt{x^2+3x-1}}\right)\right]-1}$$

$= -2$

$$= \frac{3}{\frac{1}{2}(-2)-1}$$

$$= \frac{3}{-1-1} = -\frac{3}{2}$$