Ex: Consider $f(x)=x^3-2x^2+x-1$

on interval $[\overset{a}{-1},\overset{b}{3}]$.

Find all c that satisfy MVT.

$$f(3)=3^3-2\cdot3^2+3-1$$
$$=27-18+3-1$$
$$=11$$

$$f(-1)=(-1)^3-2(-1)^2+(-1)-1$$
$$=-1-2-1-1$$
$$=-5$$

Soln: Left-side of MVT:

$$\frac{f(b)-f(a)}{b-a}=\frac{f(3)-f(-1)}{3-(-1)}$$

$$=\frac{11-(-5)}{4}$$

$$=\frac{16}{4}=4$$

line thru $(-1,-5)$ w/ slope 4

$$y+5=4(x+1)$$
$$\boxed{y=4x-1}$$

MVT says there is a c in $[-1,3]$ such that $f'(c)=4$.

So compute

$$\boxed{f'(x)=3x^2-4x+1\overset{\text{set}}{=}4}$$

$$3x^2-4x-3=0$$

$$x=\frac{-(-4)\pm\sqrt{(-4)^2-4(3)(-3)}}{2(3)}$$

$$=\frac{4}{6}\pm\frac{\sqrt{16+36}}{6}$$

$$=\frac{2}{3}\pm\frac{\sqrt{52}}{6}$$

⊕ $\quad$ ⊖

1.86 $\quad$ -0.53

$$\begin{array}{r}36\\16\\\hline52\end{array}$$

x_{11}

$$\frac{2}{3}+\frac{\sqrt{52}}{6}$$
$$\downarrow$$
$$f(x)=0.409$$
$$\Downarrow$$

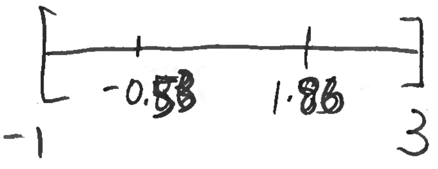
tan line at $\left(\frac{2}{3}+\frac{\sqrt{52}}{6}, 0.409\right)$ is

$$y-0.409=4\left(x-\left(\frac{2}{3}+\frac{\sqrt{52}}{6}\right)\right)$$

$$x=\frac{2}{3}-\frac{\sqrt{52}}{6}$$
$$\downarrow$$
$$f(x)=-2.261$$
$$\downarrow$$

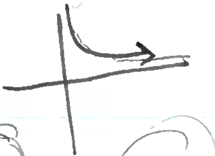
tan line at $\left(\frac{2}{3}-\frac{\sqrt{52}}{6}, -2.261\right)$ is

$$y+2.261=4\left(x-\left(\frac{2}{3}-\frac{\sqrt{52}}{6}\right)\right)$$



Limits at ± ∞ and other things

Most basic

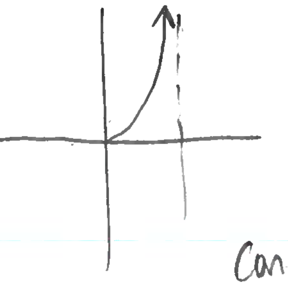


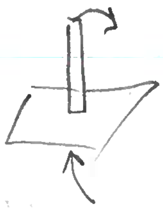
$$\lim_{x\to\infty} \frac{1}{x} = 0$$

as x gets really big

how big does $\frac{1}{x}$ get?

" $\frac{1}{\infty} = 0$ "





Control theory

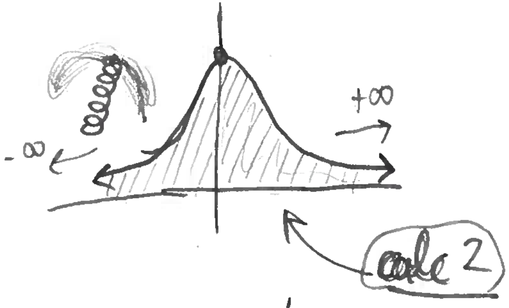
↓

stability

infinite sums

Similarly,

$$\lim_{x\to-\infty} \frac{1}{x} = 0$$



Why do $\lim_{x\to\pm\infty} f(x)$?

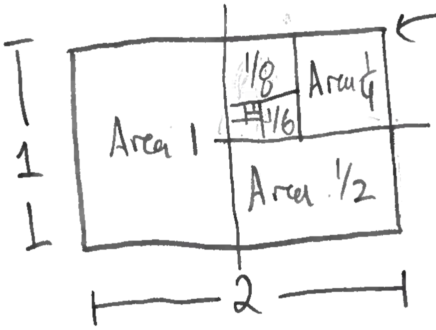
This tells you how a system evolves in long time periods.

Infinite sums

$1 + 1 + 1 + 1 + \dots \longrightarrow$ goes to ∞ "diverges"

$$\longrightarrow 1 + \frac{1}{2} + \left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^3 + \left(\frac{1}{2}\right)^4 + \dots = 2$$

really studied in calc 2



Area = 2

$$\lim_{n\to\infty} 1 + \frac{1}{2} + \left(\frac{1}{2}\right)^2 + \dots + \left(\frac{1}{2}\right)^n$$

Ex: What is

$$\lim_{x\to\infty} \frac{x^2+3x+2}{x^2-x+1} \;?$$

6

essentially comparing growth rates

$$= \lim_{x\to\infty} \left(\frac{x^2+3x+2}{x^2-x+1}\right)\left(\frac{1/x^2}{1/x^2}\right)$$

$$= \lim_{x\to\infty} \left(\frac{1+\overset{\nearrow 0}{\frac{3}{x}}+\overset{\nearrow 0}{\frac{2}{x^2}}}{1-\underset{\downarrow 0}{\frac{1}{x}}+\underset{\downarrow 0}{\frac{1}{x^2}}}\right)$$

$$= \frac{1+0+0}{1+0+0} = 1$$