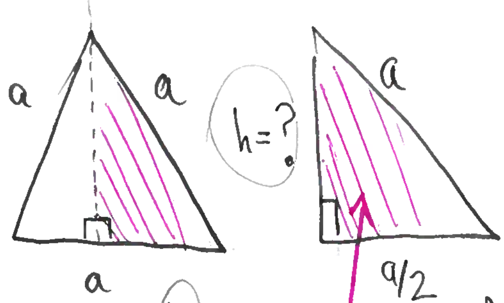
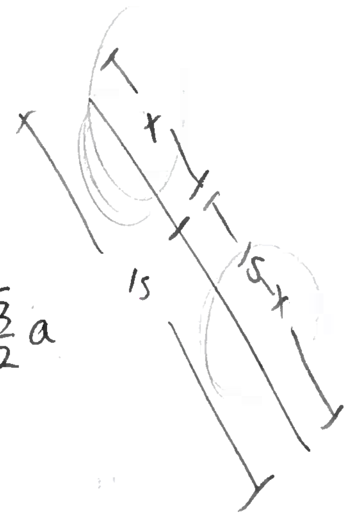




$$\frac{1}{2}\left(\frac{a}{2}\right)h = \text{Area}_{\text{right}} \longrightarrow \text{Area} = \frac{1}{2}\left(\frac{a}{2}\right)\frac{\sqrt{3}}{2}a$$

$$= \frac{\sqrt{3}}{8}a^2$$

$$\left(\frac{a}{2}\right)^2 + h^2 = a^2$$

$$h = \sqrt{a^2 - \frac{a^2}{4}} = \frac{\sqrt{3}}{2}a$$

$$\text{Area}_{\text{equilateral}} = 2\,\text{Area}_{\text{right}} = 2\frac{\sqrt{3}}{8}a^2 = \frac{\sqrt{3}}{4}a^2$$

Mean Value Theorem

relates "average values" to derivatives

Recall: if you travel 30 miles in 2 hours, then you had an average speed of $\boxed{\frac{30\text{ mi}}{2\text{ hr}} = 15\frac{\text{mi}}{\text{hr}}}$

says nothing about speeds you went over whole trip

$60\text{ min} = 1\text{ hr}$
$15\text{ min} = \frac{1}{4}\text{ hr}$
$1\text{ min} = \frac{1}{60}\text{ hr}$

Ways that could have happened?

$60\frac{\text{mi}}{\text{hr}}$ for $\boxed{15\text{ min}}$ $\rightarrow$ I go $\boxed{\frac{1}{4}60\text{ miles} = 15\text{ miles}}$

$10\frac{\text{mi}}{\text{hr}}$ for $\boxed{60\text{ min}}$ $\rightarrow$ I go 10 miles

$\frac{3}{20}\frac{\text{mi}}{\text{hr}}$ $\boxed{45\text{ min}}$ $\rightarrow$ 5 miles

$\frac{3/4}{5} =$

Compare "average speed" to "instanteneous speed"

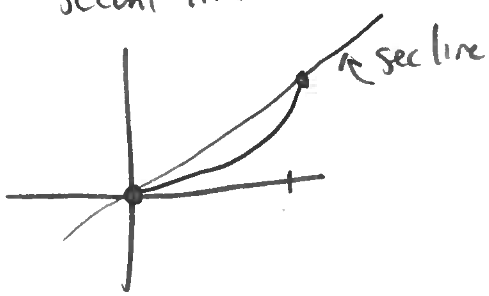
estimate based only on how long it took & how far you went

actual reading on car's speedometer

do these relate?

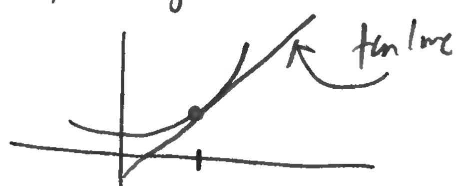
"computing average"

slope of "secant line"

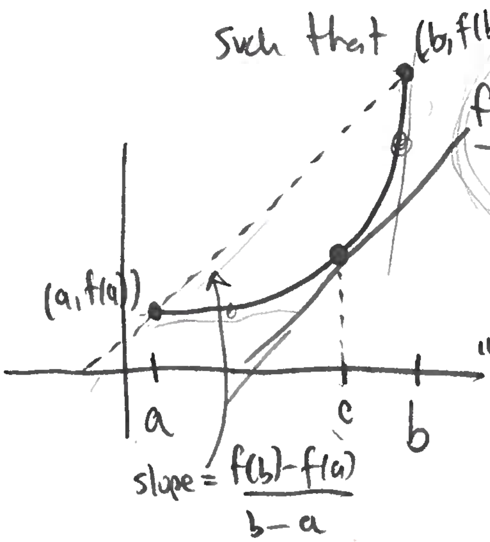


"derivative function"

slope of "tangent line"



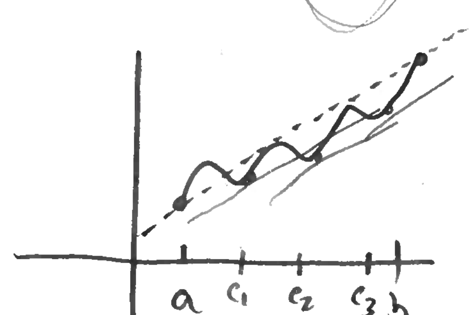
Mean Value Theorem: If $f:[a,b]\to\mathbb{R}$ is continuous and is differentiable on (a,b), then there exists a number c in (a,b) such that

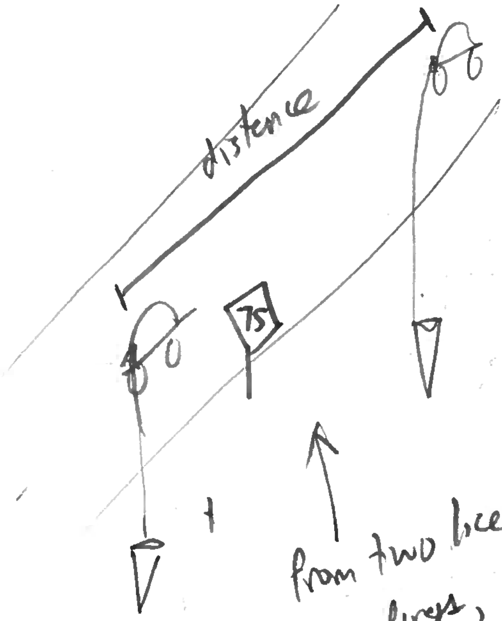


$$\frac{f(b)-f(a)}{b-a} = f'(c).$$

"average speed"

"a value of the speedometer at time c"





from two license plate readings, could find an average speed of a vehicle

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Ex : Spz that $3 \le f'(x) \le 8$ for all x.

Use MVT to find values for

$$\_\_\_ \le f(10) - f(-2) \le \_\_\_$$

Soln :

$$f(b) - f(a)$$

$$b = 10 \qquad a = -2$$

$$b - a = 10 - (-2) = 12$$

$$\_\_\_ \le \frac{f(10) - f(-2)}{12} \le \_\_\_$$

$$12 \times 8 = 96$$

b−a

$$3 \le \frac{f(10) - f(-2)}{12} \le 8 \xrightarrow{\text{mult } 12} \boxed{36 \le f(10) - f(-2) \le 96}$$

At 4:00 PM $\rightarrow$ speedometer measures $40 \frac{mi}{hr}$

4:30 PM $\rightarrow$ ———————— $80 \frac{mi}{hr}$

Question: Find an acceleration the car must have achieved.

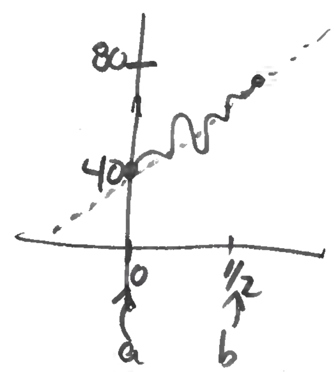
Recall: $f(x)$ miles, $x \sim hr$

$f'(x)$ $\frac{mi}{hr}$ (velocity)

$f''(x)$ $\frac{mi}{hr^2}$ (acceleration)

Soln: We are told for some distance function f, if x measures hours since 4:00 PM,

$f'(0) = 40$ at 4:00 PM $\qquad f'(1/2) = 80$ at 4:30 PM



Average acceleration:

$$\frac{f'(1/2) - f'(0)}{1/2 - 0} = \frac{80-40}{1/2} = \frac{40}{1/2} = 80 \frac{mi}{hr^2}$$

By MVT, there ~~have~~ must have been a time c b/w $x=0$ and $x=1/2$ such that

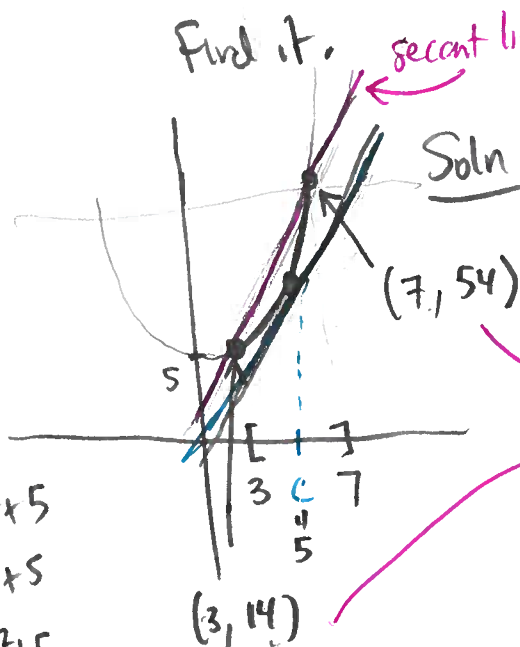
$$f''(c) = 80 \frac{mi}{hr^2}$$

EX: $f(x) = x^2 + 5$ interval $[3, 7]$

① Find average of f on this int

② By MVT we know there is some c in $[3,7]$ for which $f'(c)$ = value found above.

Find it.



Soln ①:

$f(3) = 3^2 + 5$
$= 9 + 5$

$f(7) = 7^2 + 5$
$= 49 + 5$

slope secant line $= \dfrac{54-14}{7-3} = \dfrac{40}{4} = 10$

Soln ②: We must find c such that

$$f'(c) = 10$$

$$f'(x) = 2x$$

$$f'(c) = 10$$

$$2c = 10 \rightarrow \boxed{c = 5}$$