Ex: Spz # of widgets a company can produce per hour, W, given a certain number of workers N is given by

$W|_{N=2} = 30\ln(22) + 10$

$$W = 30\ln(N+20) + 5N \quad h$$

Cost of each worker is their hourly wage, so total labor cost is

$$C = hN.$$

If the price of each widget is \$10, find profit maximizing wage (h) as a function of N.

Soln: We produce 10W money's worth of widgets. But it costs me hN to produce that amount.

$$P = \text{Profit} = 10W - hN$$

$\frac{d}{dN}\ln(N) = \frac{1}{N}$

\$\$ from producing widgets

\$\$ paid to workers

$$= 10\left[30\ln(N+20) + 5N\right] - hN$$

"profit" / worker

$$\frac{dP}{dN} = \boxed{\frac{300}{N+20} + 50 - h \overset{\text{set}}{=} 0}$$ solve for N

$$\frac{h-50}{300} = \frac{1}{N+20} \longrightarrow N+20 = \frac{300}{h-50}$$

$$\boxed{N = \frac{300}{h-50} - 20}$$ crit pt

$$\frac{d^2P}{dN^2} = 300\frac{d}{dN}(N+20)^{-1}$$

$$= \frac{-300}{(N+20)^2} < 0 \quad (\text{for all } N)$$

$\Rightarrow$ my crit pt of $\boxed{N = \frac{300}{h-50} - 20}$

is where a local max of P occurs

$\Rightarrow$ the optimal number of workers is

$$N = \frac{300}{h-50} - 20$$

$\downarrow$ solve for h

$$\frac{1}{N} = \frac{h-50}{300} \rightarrow \frac{300}{N} = h-50$$

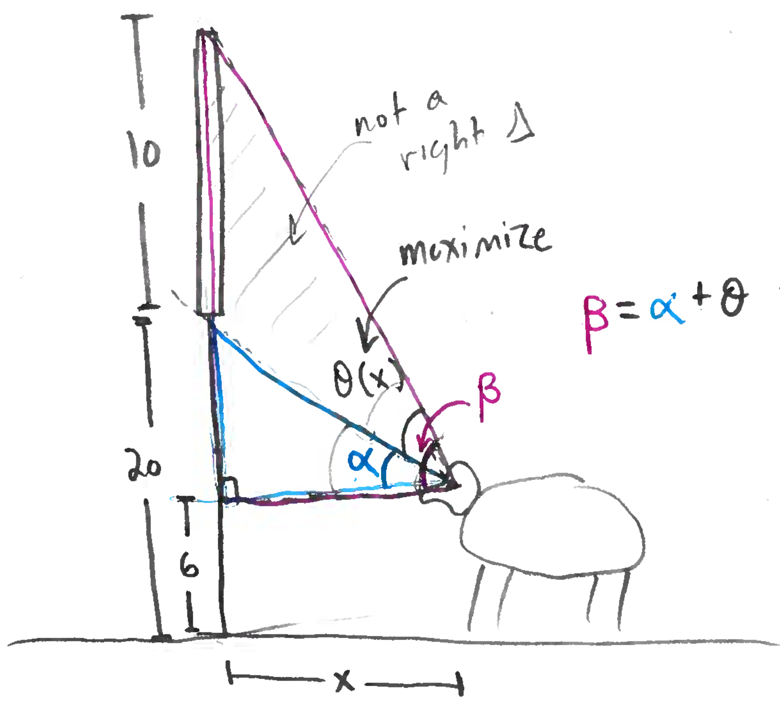
$$\rightarrow h = 50 + \frac{300}{N}$$

Ex: A rectangular billboard 10 ft in height stands in a field so its bottom is 20 ft from ground.

A cow w/ eye level at 6 ft above ground stands x ft from billboard.

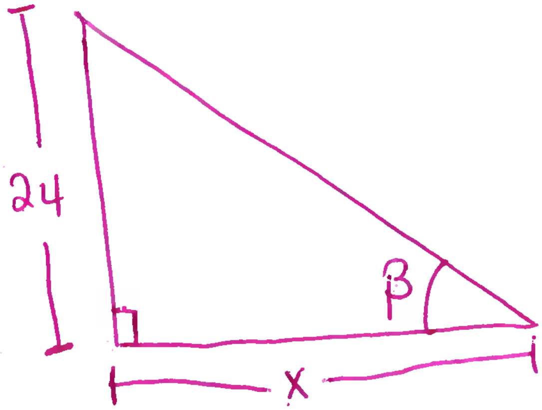
Express θ — vertical angle subtended by billboard at cow's eye in terms of x.

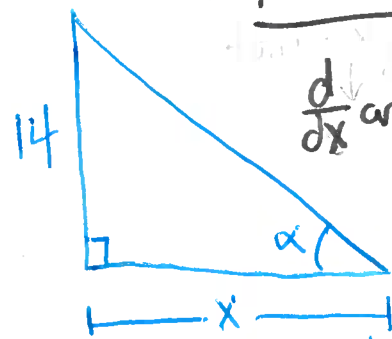
Then find distance x_0 the cow must stand to maximize θ.



maximize as a function of x

$$\beta = \alpha + \theta \Rightarrow \boxed{\theta = \beta - \alpha} \quad (\star)$$





Recall

$$\frac{d}{dx}\arctan(x) = \frac{1}{1+x^2}$$

$$\tan(\beta) = \frac{24}{x}$$

$$\beta = \arctan\left(\frac{24}{x}\right)$$

$$\tan(\alpha) = \frac{14}{x}$$

$$\alpha = \arctan\left(\frac{14}{x}\right)$$

So by (★),

$$\theta(x) = \beta - \alpha = \arctan\left(\frac{24}{x}\right) - \arctan\left(\frac{14}{x}\right)$$

$$\Rightarrow \theta'(x) = \frac{d}{dx}\arctan\left(\frac{24}{x}\right) - \frac{d}{dx}\arctan\left(\frac{14}{x}\right)$$

mismatch

$\frac{1}{x} = x^{-1}$

$$= \frac{d(24/x)}{dx}\,\frac{d}{d(24/x)}\arctan\left(\frac{24}{x}\right) - (\text{similar})$$

$$= \frac{\frac{-24}{x^2}}{1+\left(\frac{24}{x}\right)^2} - \frac{\frac{-14}{x^2}}{1+\left(\frac{14}{x}\right)^2}$$

multiply by $1 = \frac{x^2}{x^2}$

$$= \boxed{\frac{-24}{(x^2+24^2)} + \frac{14}{(x^2+14^2)} \overset{\text{set}}{=} 0}$$

$$-24(x^2+14^2) + 14(x^2+24^2) = 0$$

$$-10x^2 = (24)(14^2) - 14(24^2) = -3360$$

$$x^2 = \frac{-3360}{-10} = 336$$

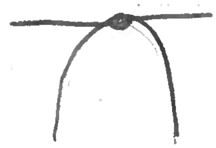
neg soln not phys meaningful

$$x = \pm\sqrt{336}$$

c.p.

$$\theta''(x) = \frac{d}{dx}\left[\frac{-24}{x^2+24^2} + \frac{14}{x^2+14^2}\right]$$

$$= \frac{24(2x)}{(x^2+24^2)^2} - \frac{14(2x)}{(x^2+14^2)}$$



$$\theta''(\sqrt{336}) = -0.00076 < 0$$

c.p.

$\Rightarrow$ by 2nd deriv test, maximum of 0.26 rad

at $x = \sqrt{336}$

14.9°