Ex: Current, I, in amps, is given by

$$I = \cos(wt) + 2\sin(wt)$$

where $w \neq 0$ is a constant.

What are the maximum & minimum values of I?

Soln: Restrict attention to $\boxed{0 \leq wt \leq 2\pi} \xleftrightarrow{\text{div by } w} 0 \leq t \leq \frac{2\pi}{w}$

Find crit pts:

$$\frac{dI}{dt} = -w\sin(wt) + 2w\cos(wt) \overset{\text{set}}{=} 0$$

$\sin(2t) = \frac{1}{2}$

$$\Downarrow$$

$$2w\cos(wt) = w\sin(wt)$$

$$\cos(wt) = \frac{w\sin(wt)}{2w} = \frac{1}{2}\sin(wt)$$

Let $\boxed{\psi = wt.}$

$$\cos(\psi) = \frac{1}{2}\sin(\psi)$$

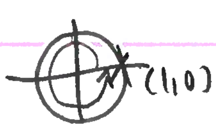
Desmos $\longrightarrow$ $\psi = 1.107, 4.249$

$wt = 1.107 \longrightarrow t = \frac{1.107}{w}$

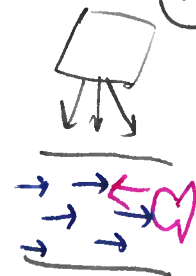
$wt = 4.249 \longrightarrow t = \frac{4.249}{w}$

Find extreme values:

t	$I(t) = \cos(wt) + 2\sin(wt)$
0	$I(0) = \cos(0) + 2\sin(0) = 1$
$\frac{2\pi}{w}$	$I(\frac{2\pi}{w}) = \cos(\cancel{w}\cdot\frac{2\pi}{\cancel{w}}) + 2\sin(w\cdot\frac{2\pi}{w}) = 1$
$\frac{1.107}{w}$	2.23 ← max value of I
$\frac{4.249}{w}$	-2.23 ← min val of I



Ex: For fish swimming at speed v relative to water, its energy expenditure is proportional to v^3.



Belief: fish minimize energy to swim — Scientific theory

If fish swim against a current u ($u < v$) $\rightarrow$ $0 < v-u$

then the time required to swim distance L is $\frac{L}{v-u}$ and total energy required is

$$\boxed{E(v) = av^3 \frac{L}{v-u}}$$

where a is a constant.

Find value of v that minimizes E. $\leftarrow$ tells us what to compare to when doing experiment

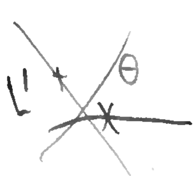
Soln: $\frac{dE}{dv} = aL \frac{d}{dv}\left[v^3 \cdot \frac{1}{v-u}\right]$ $(v-u)^{-1}$ chain rule

(PROD RULE)

$$\frac{d}{dx}[5f(x)] = 5\left[\frac{d}{dx}f(x)\right]$$

$$= aL\left[3v^2 \frac{1}{v-u} + v^3\left(-(v-u)^{-2}(1)\right)\right]$$

$$\boxed{= aL\left[\frac{3v^2}{v-u} - \frac{v^3}{(v-u)^2}\right] \overset{\text{set}}{=} 0}$$



Div by aL + get common denom

$$\frac{3v^2(v-u) - v^3}{(v-u)^2} = 0 \xrightarrow{\text{mult by } (v-u)^2} 3v^2(v-u) - v^3 = 0$$

$(v-u)^2 \neq 0$ because

$$v^2[3v - 3u - v] = 0$$

$\cancel{v=0}$ $\leftarrow$ physically meaningless — means fish are not moving

$3v - 3u - v = 0$

$2v = 3u \rightarrow \boxed{v = \frac{3u}{2}}$

should be the answer BUT need to verify it gives a min

Recall 2nd deriv test:

$f'' > 0$ con↓, $f' = 0$

⇓

max

$f'' < 0$ $f' = 0$

con↑

⇓

min

Need

$$\frac{d^2E}{dv^2} = \frac{d}{dv}\frac{dE}{dv} = \frac{d}{dV}\left[aL\left(\frac{3v^2}{v-u} - \frac{v^3}{(v-u)^2}\right)\right]$$

quotient rule twice

$$= aL\left[\frac{(v-u)6v - 3v^2(1)}{(v-u)^2} - \left[\frac{(v-u)^2 3v^2 - v^3 2(v-u)}{(v-u)^4}\right]\right]$$

dist 1

$$= aL\left[\frac{(v-u)^2\left[(v-u)6v - 3v^2\right] - (v-u)^2 3v^2 + 2v^3(v-u)}{(v-u)^4}\right]$$

$$= aL\left[\frac{\cancel{(v-u)}\left[(v-u)^2 6v - 3v^2(v-u) - (v-u)3v^2 + 2v^3\right]}{(v-u)^{\cancel{4}}}\right]$$

$$= aL\left[\frac{(v-u)^2 6v - 6v^2(v-u) + 2v^3}{(v-u)^3}\right]$$

Now at crit pt:

$$\left.\frac{d^2E}{dv^2}\right|_{v=\frac{3u}{2}} = aL\left[\frac{\left(\frac{u}{2}\right)^2(9u) - 6\left(\frac{9u^2}{4}\right)\left(\frac{u}{2}\right) + 2\left(\frac{27u^3}{8}\right)}{\left(u/2\right)^3}\right]$$

$\frac{3u}{2} - u = \frac{3u}{2} - \frac{2u}{2}$

$6v = 6\left(\frac{3u}{2}\right) = 9u$

$$= aL\left[\frac{u^3\left[\frac{9}{4} - \frac{6\cdot 9}{8} + \frac{27}{4}\right]8}{u^3}\right] = 18aL > 0$$

⇓ proves

MINIMUM

OHW8 P1

$$f = e^{5x} + e^{-x} \qquad (-\infty, 3]$$

$$\ln(ab) = \ln(a) + \ln(b)$$

$$\cancel{\ln}(e^{☺}) = ☺$$

$\downarrow$

$$f' = \boxed{5e^{5x} - e^{-x} \overset{\text{set}}{=} 0}$$

$$5e^{5x} = e^{-x}$$

$\downarrow \ln$

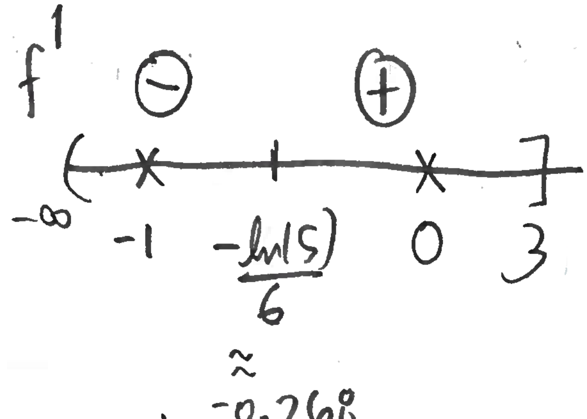
$$\ln(5e^{5x}) = \ln(e^{-x})$$

$$\ln(5) + \ln(e^{5x}) = \ln(e^{-x})$$

$$\ln(5) + 5x = -x$$

$$6x = -\ln(5)$$

$$x = -\frac{\ln(5)}{6}$$



$\Downarrow$

decreasing on $\left(-\infty, -\frac{\ln(5)}{6}\right)$

inc on $\left(-\frac{\ln(5)}{6}, 3\right)$

open ints