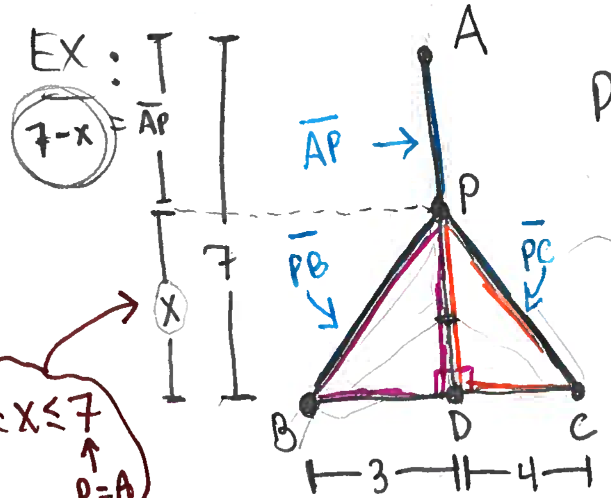
EX:



Point P needs to be located on $\overline{AD}$ so the total length of cables linking P to A, B, C is minimized.

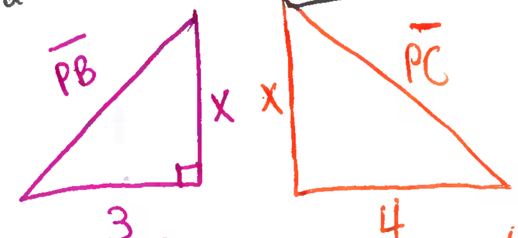
$0 \le x \le 7$

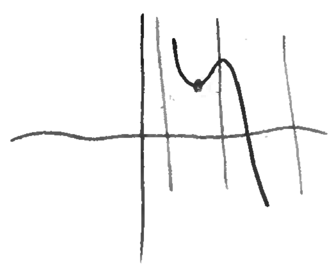
P on bottom

P = A

GOAL: minimize

need to express in terms of a variable $\longrightarrow$ $\boxed{L = \overline{AP} + \overline{PB} + \overline{PC}}$





$3^2 + x^2 = (\overline{PB})^2$

$\overline{PB} = \pm\sqrt{9+x^2}$

$4^2 + x^2 = (\overline{PC})^2$

$\overline{PC} = \sqrt{16+x^2}$

$$\Rightarrow \quad L = (7-x) + \sqrt{9+x^2} + \sqrt{16+x^2}$$

$$\frac{dL}{dx} = \boxed{-1 + \frac{x}{\sqrt{9+x^2}} + \frac{x}{\sqrt{16+x^2}} \overset{\text{set}}{=} 0}$$

calculator

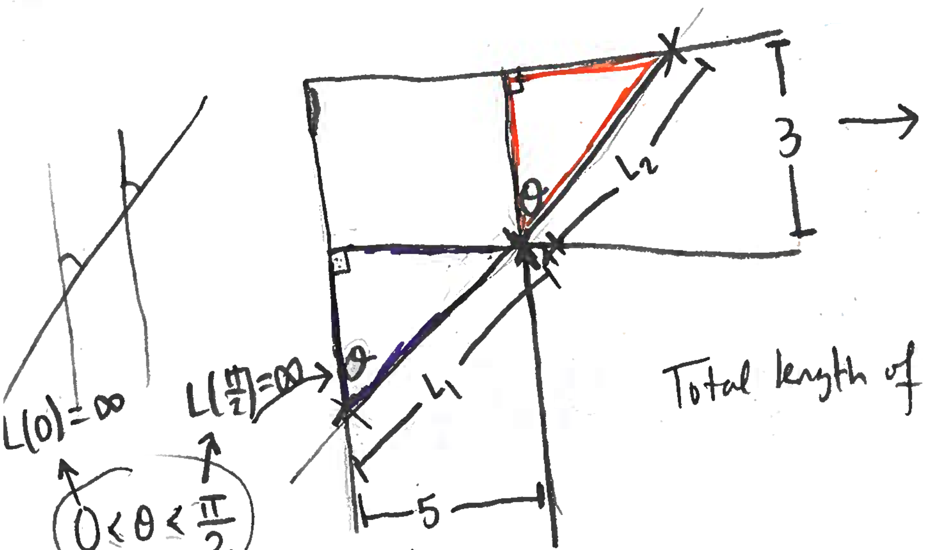
$\boxed{x \approx 1.995}$ ← critical pt

x	L
0	14
1.995	13.078
7	15.678

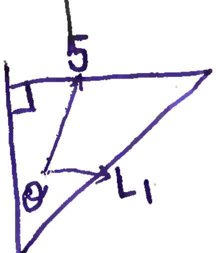
absolute min of 13.078 at x = 1.995

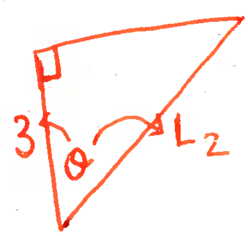
Ex: Pipe carried down a 5ft wide hallway. At end of hall, right angled turn into hallway that is 3ft wide.

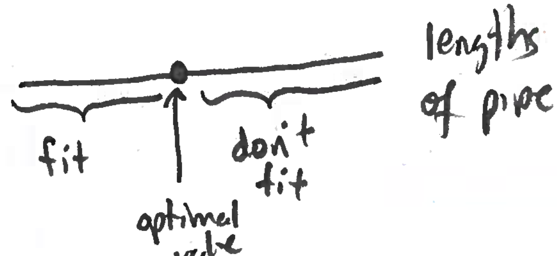
What is length of largest pipe that can be carried around corner?



Total length of pipe $= L = L_1 + L_2$







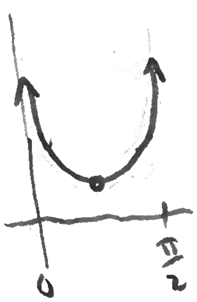
$\sin(\theta) = \frac{5}{L_1}$

$L_1 = \frac{5}{\sin(\theta)}$

$\cos(\theta) = \frac{3}{L_2}$

$L_2 = \frac{3}{\cos(\theta)}$

- largest pipe that fits
- shortest pipe that doesn't fit



$$\Rightarrow L = \frac{5}{\sin(\theta)} + \frac{3}{\cos(\theta)}$$

$$\frac{dL}{d\theta} = \frac{\sin(\theta)\frac{d}{d\theta}[5] - 5\frac{d}{d\theta}[\sin\theta]}{\sin^2(\theta)} + \frac{-3\frac{d}{d\theta}\cos(\theta)}{\cos^2(\theta)}$$

$-\sin(\theta)$

0

$$= \boxed{\frac{-5\cos(\theta)}{\sin^2(\theta)} + \frac{3\sin(\theta)}{\cos^2(\theta)} \overset{\text{set}}{=} 0}$$

$$\downarrow$$

$$\frac{3\sin(\theta)}{\cos^2(\theta)} = \frac{5\cos(\theta)}{\sin^2(\theta)}$$

$$3\sin^3(\theta) = 5\cos^3(\theta)$$

$$\tan^3(\theta) = \frac{\sin^3(\theta)}{\cos^3(\theta)} = \frac{5}{3}$$

$$\tan^3(\theta) = \frac{5}{3}$$

$$\tan(\theta) = \sqrt[3]{\frac{5}{3}}$$

$$\theta = \arctan\left(\sqrt[3]{\frac{5}{3}}\right) \approx 0.8701 \text{ rad}$$

$\frac{dL}{d\theta}$ ⊖   ⊕

0   0.7   0.8701   0.9 $\frac{\pi}{2} \approx 1.5\ldots$

$\Downarrow$

$\theta = 0.8701$ corresponds to the optimal value

Thus the length we want is

$$\boxed{L(0.8701) = 11.19 \text{ ft}}$$

$\uparrow$

technically: need a pipe length < $\cancel{\frac{5}{\sin(\text{arct}\ldots)}}$