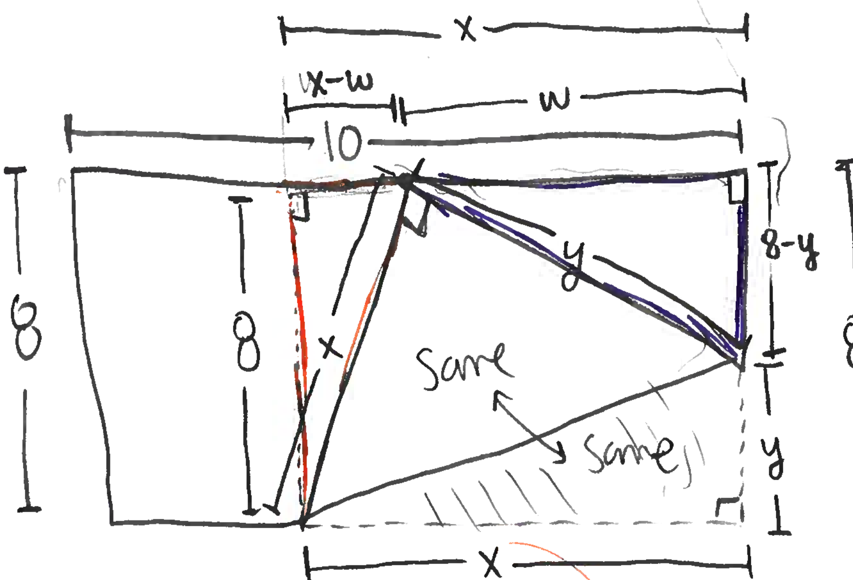


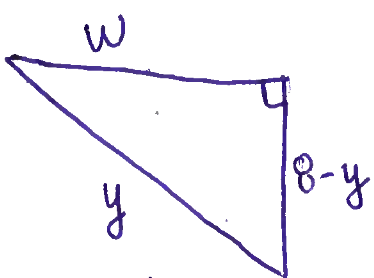
$$\boxed{\text{Area} = \tfrac{1}{2}xy}$$

extreme values

$$\boxed{0 < x \le 10}$$

$$\boxed{0 < y \le 8}$$

$\boxed{\text{must consider at end}}$



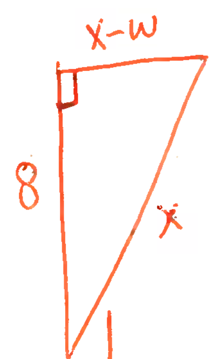
Pythag theorem

$$\boxed{y^2 = w^2 + (8-y)^2}$$

$$\cancel{y^2} = w^2 + 64 - 16y + \cancel{y^2}$$

$$16y = w^2 + 64$$

$$\boxed{y = \frac{w^2+64}{16}}$$



$$\boxed{8^2 + (x-w)^2 = x^2}$$

$$64 + \cancel{x^2} - 2xw + w^2 = \cancel{x^2}$$

$$64 + w^2 = 2xw$$

$$\boxed{x = \frac{64+w^2}{2w}}$$

express x and y in terms of w only

$$\text{Area} = \frac{1}{2}\underbrace{\left[\frac{w^2+64}{2w}\right]}_{\text{"x"}}\left[\frac{w^2+64}{16}\right]$$

$$= \frac{(w^2+64)^2}{64w}$$

$$\frac{d\,\text{Area}}{dw} = \frac{64w\left(2(w^2+64)(2w)\right) - (w^2+64)^2\,64}{(64w)^2}$$

②

$$\frac{d\text{Area}}{dw} = \frac{\cancel{64}(w^2+64)\Big[\overbrace{w(4w)-(w^2+64)}^{4w^2-w^2-64}\Big]}{(64w)^2}$$

$$= \boxed{\frac{(w^2+64)\left[3w^2-64\right]}{64w^2} \text{ set } = 0}$$

can't have $w=0$
b/c div by zero

mult by $64w^2$

$$(w^2+64)(3w^2-64)=0$$

$$w^2+64=0 \qquad\qquad 3w^2-64=0$$

$$\cancel{w=\pm\sqrt{-64}}$$
$$\cancel{=\pm 8i}$$

~~throw out~~
~~not physically~~
~~meaningful~~

$$3w^2=64$$

$$w^2=\frac{64}{3}$$

$$w=\pm\sqrt{\frac{64}{3}}=\cancel{\pm}\frac{8}{\sqrt{3}}$$

neg soln not phys meaningful

only one crit pt:

$$\boxed{w=\frac{8}{\sqrt{3}}}$$

Check Area $= \dfrac{(w^2+64)^2}{64w}$

at crit pt: $w = \dfrac{8}{\sqrt{3}}$

and at extreme values: $\begin{bmatrix} x=10 \\ y=8 \end{bmatrix}$ → need to translate into w's

We know

$$x = \frac{64+w^2}{2w}$$

$$y = \frac{w^2+64}{16}$$

$\underline{x=10}$

$$10 = \frac{64+w^2}{2w}$$

$$20w = 64+w^2$$

$$w^2-20w-64=0$$

↓ QF

$$w = \frac{-(-20) \pm \sqrt{(-20)^2 - 4(1)(-64)}}{2(1)}$$

$$= \frac{20}{2} \pm \frac{\sqrt{656}}{2}$$

→ $\boxed{w = 22.80}$ ← check area

→ $\cancel{w = -2.8}$ not phys meaningful

$\underline{y=8}$

$$8 = \frac{w^2+64}{16}$$

$$128 = w^2+64$$

$$64 = w^2$$

$\boxed{w = \pm 8 \rightarrow +8}$

Finally

w	Area $= \frac{(w^2+64)^2}{64w}$
$8/\sqrt{3}$	24.63 ⟵ smallest ⟹ abmin of 24.63 at $w = 8/\sqrt{3}$
22.80	233.6 ⟵ largest ⟹ abmax of 233.6 at $w = 22.8$
8	32