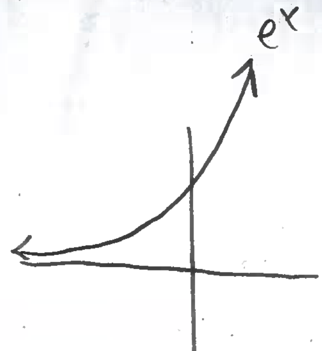
1

Ex: Find abs extrema of $f(x) = e^{x^2-2}$ over interval $[-2,2]$.



Soln: Crit pts

$\cancel{ab=2}$

$a=2 \quad b=2$

$ab=0$

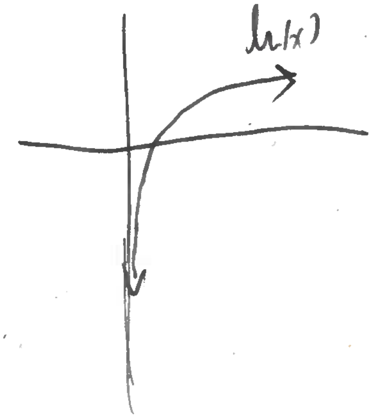
$a=0 \quad b=0$

$$\frac{d}{dx} f(x) = \frac{d}{dx} e^{x^2-2}$$

$$= \frac{d}{dx}(x^2-2)\,\frac{d}{d(x^2-2)} e^{x^2-2}$$

$$\boxed{= (2x) e^{x^2-2} \overset{\text{set}}{=} 0}$$

e^x ∅ is new zero!!



OR

$2x = 0 \rightarrow \boxed{x=0}$

$e^{x^2-2} = 0 \rightarrow$ NO SOLN

	x-vals	f(x)
Crit pt $\rightarrow$	0	$e^{0-2} = e^{-2} \approx 0.135$
endpts $\rightarrow$	-2	$e^{(-2)^2-2} = e^{4-2} = e^{2} \approx 7.39$
endpts $\rightarrow$	2	$e^{2^2-2} = e^{2} \approx 7.39$

abmin of 0.135 at $x=0$

abmax of 7.39 occurring at $x = 2, -2$

Ex: $f(x) = e^{x^2+x-2}$ over $[-2,3]$.

Find abs extrema

Soln: crit pts

$$f'(x) = \frac{d}{dx} e^{x^2+x-2}$$

$$= \frac{d(x^2+x-2)}{dx} \frac{d}{d(x^2+x-2)} e^{x^2+x-2}$$

$$= \boxed{(2x+1)e^{x^2+x-2} \overset{\text{set}}{=} 0}$$

$2x+1=0$ $\downarrow$ $x = -\frac{1}{2}$

$e^{x^2+x-2} = 0$ $\downarrow$ NO SOLN

x-values	f(x)
crit pt $\rightarrow$ $-\frac{1}{2}$	0.11 $\leftarrow$ abmin $\Rightarrow$ abmin of 0.11 at $x = -1/2$
-2	1
3	22026 $\leftarrow$ abmax $\Rightarrow$ abmax of 22026.47 at $x = 3$

Second derivative test

Spz $x=c$ is a crit pt s.t. $f'(c)=0$,

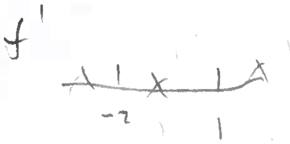
① if $f''(c)>0$, then there is a local min at $x=c$

② if $f''(c)<0$, then there is a local max at $x=c$.

Ex: $f(x) = \frac{1}{3}x^3 + \frac{1}{2}x^2 - 2x$

↓ crit pts

$f'(x) = \boxed{x^2 + x - 2 \overset{\text{set}}{=} 0}$



$(x+2)(x-1)=0$

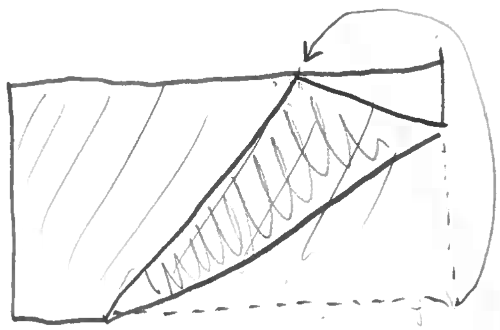
$x=-2 \quad x=1$ ← crit pts

$f''(x) = 2x+1$

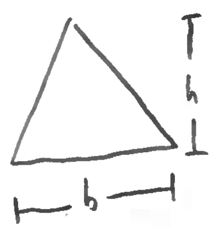
$f''(-2) = 2(-2)+1 = -3 < 0 \longrightarrow$ local max at $x=-2$

$f''(1) = 2(1)+1 = 3 \quad > 0 \longrightarrow$ local min at $x=1$

Ex: Rectangular piece of paper — length of 10 cm & height of 8 cm.

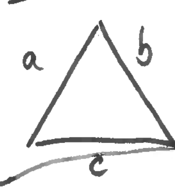


What is max & min area such a triangle can have?



$\Rightarrow$ ~~Area = $\frac{1}{2}bh$~~

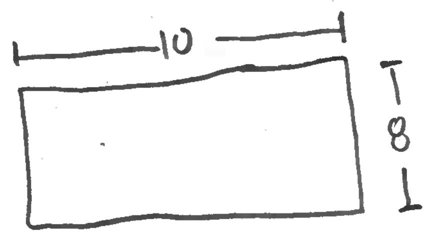
Heron's formula

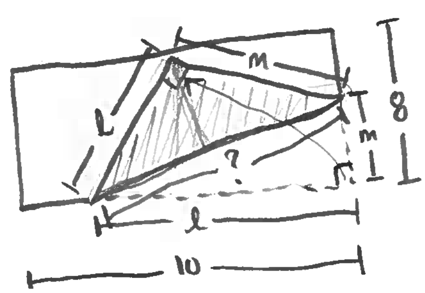


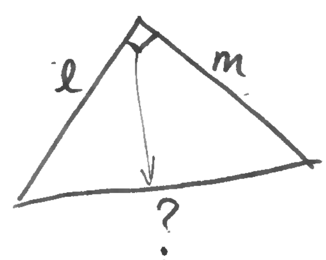
$\rightarrow s = \frac{a+b+c}{2}$

$\rightarrow$ Area $= \sqrt{s(s-a)(s-b)(s-c)}$

Soln:







Pyth thm:

$l^2 + m^2 = ?^2$

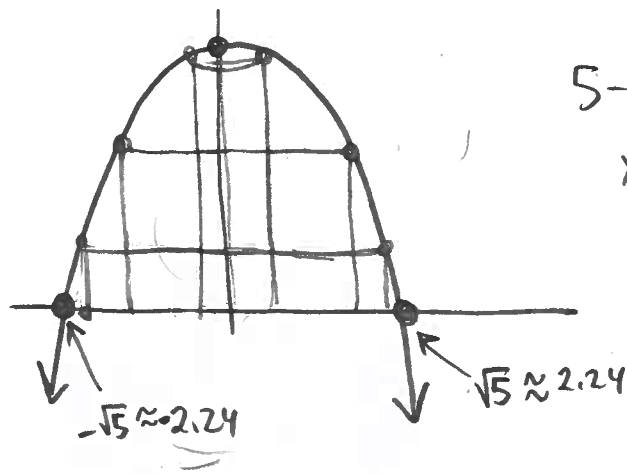
$? = \sqrt{l^2 + m^2}$

Stuck $\rightarrow$ return tomorrow

Ex: A rectangle is inscribed w/ its base on x-axis & its upper corners on parabola $y = 5 - x^2$.

What are dimensions of rectangle w/ greatest area?

Soln:



$5 - x^2 = 0$

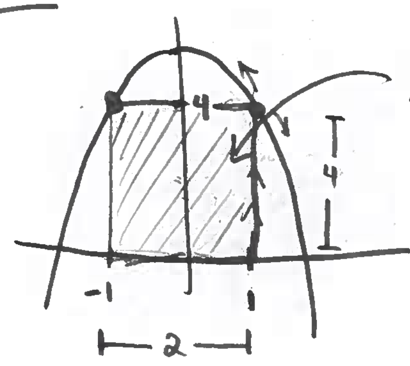
$x = \pm\sqrt{5}$

examples

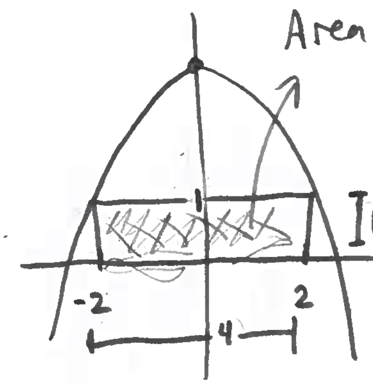
$5 - x^2 = 4$

$x^2 = 1$

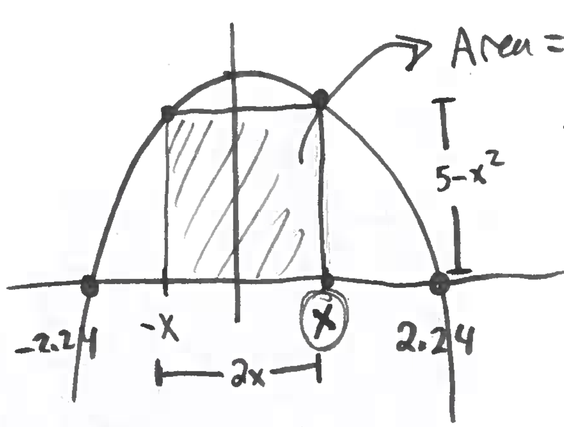
$x = \pm 1$



Area = 8



Area = 4



$\text{Area} = 2x(5 - x^2)$

$= 10x - 2x^3$

Goal : optimize $\begin{cases} \text{Area} = 2x(5-x^2) = 10x - 2x^3 \\ \text{on } [-2.24, 2.24] \end{cases}$

$$\frac{d\,\text{Area}}{dx} = 10 - 6x^2 \overset{\text{set}}{=} 0$$

$$x^2 = \frac{10}{6} = \frac{5}{3} \longrightarrow x = \pm\sqrt{\frac{5}{3}}$$

b/c in our picture "x" was positive $\rightarrow \; = \sqrt{5/3}$

x-vals	Area
endpts → -2.24	0
endpts → 2.24	0
$\sqrt{5/3}$	8.61 ← largest

$\Rightarrow$ absmax of 8.61 at $x = \sqrt{5/3}$

$$\frac{d^2\text{Area}}{dx^2} = -12x$$

at $x = \sqrt{5/3} \rightarrow \dfrac{d^2\text{Area}}{dx^2} < 0 \Rightarrow$ local max