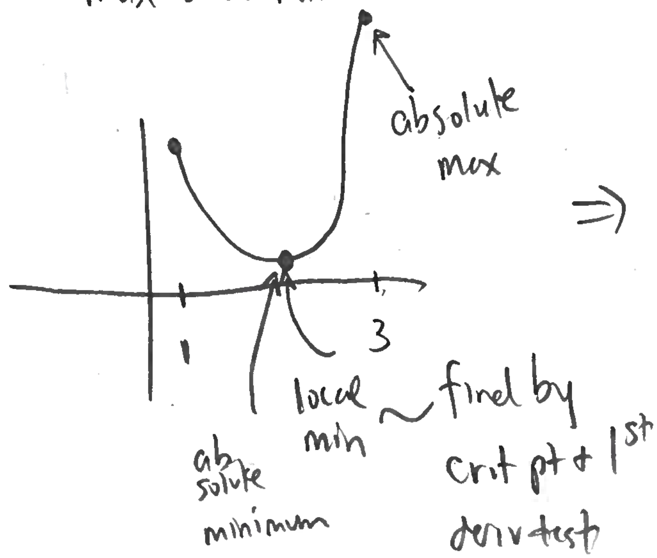
Absolute extrema

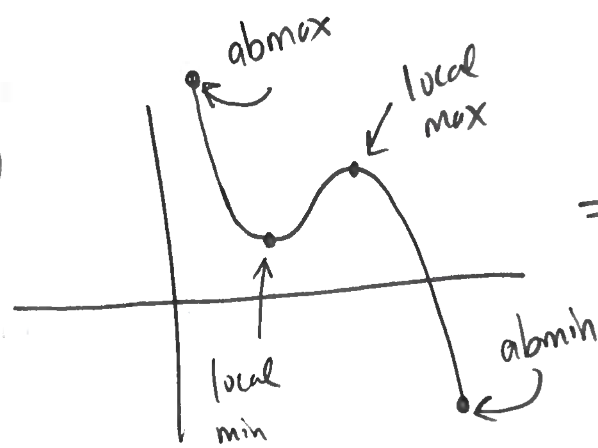
The "true" max and min

ex)



$\Rightarrow$ local extremum is absolute ✓

ex)

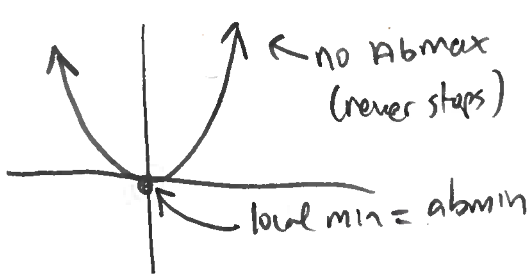


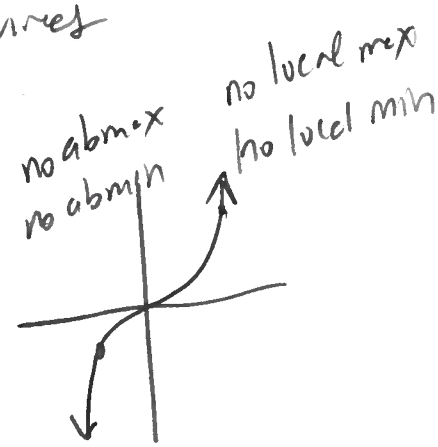
$\Rightarrow$ NO local extremum is absolute!

Conclusion: * finding local extrema may not be "good enough" sometimes

* finding absolute extrema often requires a restricted domain

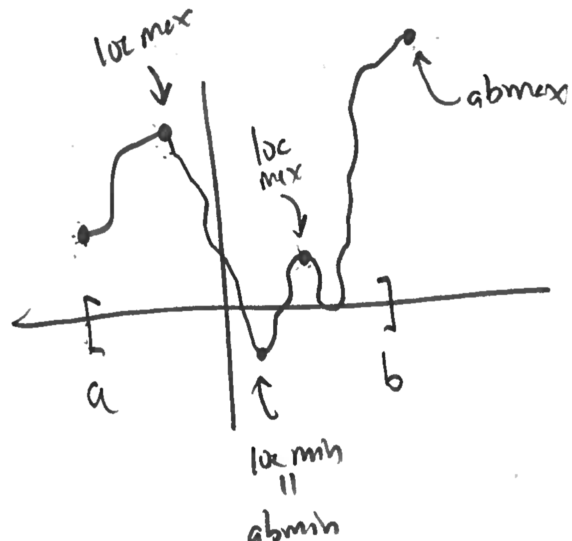
ex)





<u>Extreme Value theorem</u>: A function continuous on a closed bounded interval — $[a,b]$ — attains an absolute max and an absolute min on $[a,b]$.

ex)



<u>Algorithm</u>: if f is differentiable on $[a,b]$, then absolute extrema can be found by:

1. find crit pts $c_1, c_2, \ldots, c_n$
2. calculate $f(c_i)$ ~ f at all crit pts and $f(a)$ and $f(b)$
3. largest → ab max
   smallest → ab min

How to find absolute extrema

Ex: Find abs max & abs min of

$$f(x) = (x+2)^{3/4}$$

over interval $[-1, 5]$.

$$\frac{d}{dt} t^n = n t^{n-1}$$

Soln: find crit pts

$f' = 0$ and wherever f' not defined

$$f'(x) = \frac{d}{dx}\left((x+2)^{3/4}\right)$$

mismatch

$$= \frac{d(x+2)}{dx} \frac{d}{d(x+2)} (x+2)^{3/4}$$

$$= (1)\,\frac{3}{4}(x+2)^{3/4 - 1}$$

$$= \boxed{\frac{3}{4}(x+2)^{-1/4} \overset{\text{set}}{=} 0}$$

$$f' = \frac{1}{(x+2)^{1/4}} = 0 \rightarrow \text{no soln}$$

f' not def. when $x+2=0$

$x = -2$

not in $[-1,5]$

x-values	f(x)	
crit pt → ~~-2~~	~~$(-2+2)^{3/4} = 0$~~	~~smallest ⇒ abmin of 0 at x = -2~~
endpts → -1	$(-1+2)^{3/4} = 1^{3/4} = 1$	smallest ⇒ abmin of 1 at $x = -1$
endpts → 5	$(5+2)^{3/4} = 7^{3/4} \approx 4.30$	biggest ⇒ abmax of 4.3 at $x=5$

Ex: Find abs extrema of

$$f(x) = e^x - 3x$$

on $[-5, 5]$

$$\frac{d}{dx} e^x = e^x$$

Soln: Crit pts

$$f'(x) = \boxed{e^x - 3 \overset{\text{set}}{=} 0}$$

$$\ln(e^x) = x$$

$$e^{\ln x} = x$$

$$e^x = 3$$

$$\downarrow$$

$$\ln(e^x) = \ln(3)$$

$$\boxed{x = \ln(3)}$$

	x-vals	f(x)		
crit pt $\longrightarrow$	$\ln(3)$	-0.295	$\longleftarrow$ smallest	$\Rightarrow$ abs min of -0.295 at $x = \ln(3)$
	-5	15.00		
	5	133.41	$\longleftarrow$ largest	$\Rightarrow$ abs max of 133.41 at $x = 5$

Ex: Abs extrema of

$$f(x) = 2x^3 + 3x^2 - 2x + 1$$

on $[-15, 15]$.

Soln: crit pts

$$f' = \boxed{6x^2 + 6x - 2 \overset{\text{set}}{=} 0}$$

$$3x^2 + 3x - 1 \overset{\text{set}}{=} 0$$

9+12

$$x = \frac{-3 \pm \sqrt{3^2 - 4(3)(-1)}}{2(3)}$$

$$= -\frac{3}{6} \pm \frac{\sqrt{21}}{6}$$

$\oplus$ ≈ 0.264

$\ominus$ ≈ -1.26

x-values	f(x)
$-\frac{3}{6} + \frac{\sqrt{21}}{6}$	0.718
$-\frac{3}{6} - \frac{\sqrt{21}}{6}$	4.28
15	7396 ← largest
-15	-6044 ← smallest (most negative)

$\Longrightarrow$ abmax of 7396 at $x = 15$

$\Longrightarrow$ absmin of -6044 at $x = -15$