EX: Find where $f(x) = 2x^3 + 3x^2 - 36$ is incr/decr, local extrema, concavity, and any inflection pts. ← extrema of f'

Soln: find crit pts (of f): $f' = 6x^2 + 6x \overset{\text{set}}{=} 0$

$x(6x+6) = 0$

$x = 0$ or $6x + 6 = 0$

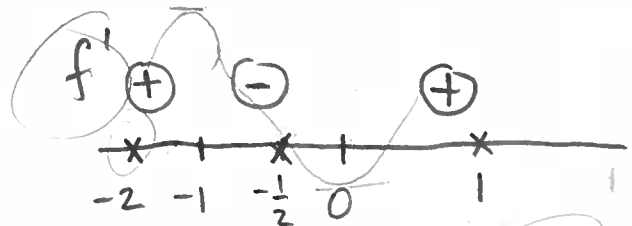
$x = -1$

potential extrema of f

find crit pts of f': $f'' = 12x + 6 \overset{\text{set}}{=} 0$

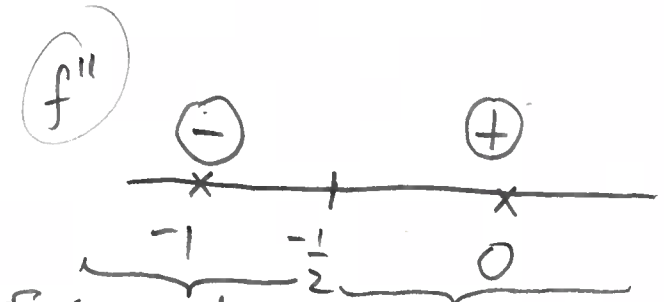
$x = -\frac{6}{12} = -\frac{1}{2}$

potential infl pt



⇒ local max of -35 at $x = -1$

local min of -36 at $x = 0$



f is concave down on $(-\infty, -\frac{1}{2})$

f is concave up on $(-\frac{1}{2}, \infty)$

infl pt at $x = -\frac{1}{2}$

Ex: Sine instructions: $\boxed{f(x) = \sin(x) + \cos(x) \quad 0 \le x \le 2\pi}$

Soln: crit pts of f

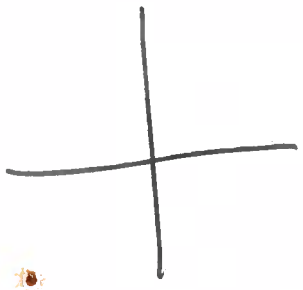
$f' = \cos(x) - \sin(x) \overset{\text{set}}{=} 0$

$\Downarrow$

$\cos(x) = \sin(x)$

$\Downarrow$ unit circle

$x = \frac{\pi}{4}, \frac{5\pi}{4}$



crit pts of f'

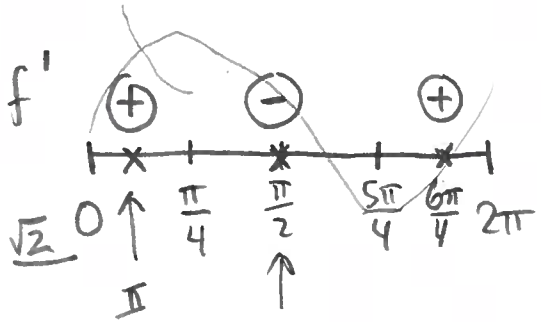
$f'' = -\sin(x) - \cos(x) \overset{\text{set}}{=} 0$

$-\sin(x) = \cos(x)$

$\Downarrow$ unit circle

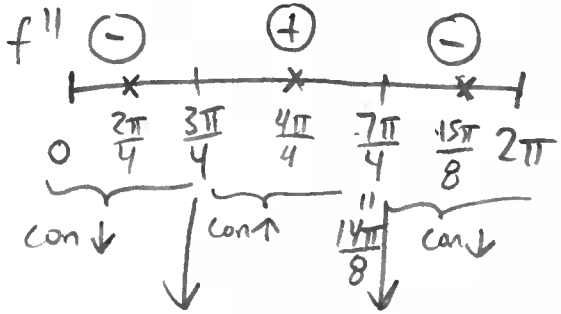
$x = \frac{3\pi}{4}, \frac{7\pi}{4}$

$f''(\pi) = 0 - (-1) = 1$



$\Rightarrow$ local max of $\sqrt{2}$ at $x = \frac{\pi}{4}$

and local min of $-\sqrt{2}$ at $x = \frac{5\pi}{4}$



infl pts at: $\frac{3\pi}{4}$, $\frac{7\pi}{4}$