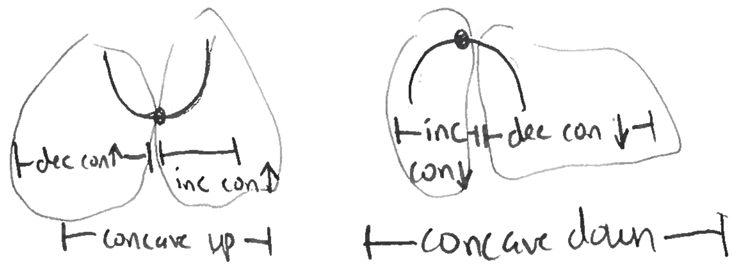
Earlier: we learned that $f'(t)>0$ then f is increasing at t

+ if $f'(t)<0$ then f is decreasing at t

Goal: understand optimization

Concavity – way a curve is shaped



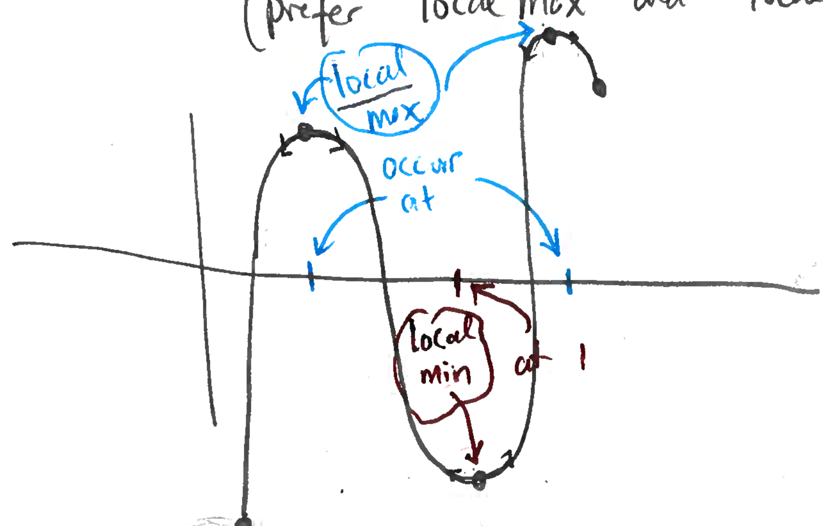
TURNS OUT – concavity is entirely determined by 2nd derivative (derivative of derivative)

$f''(t)>0 \Longrightarrow$ f is concave up at t

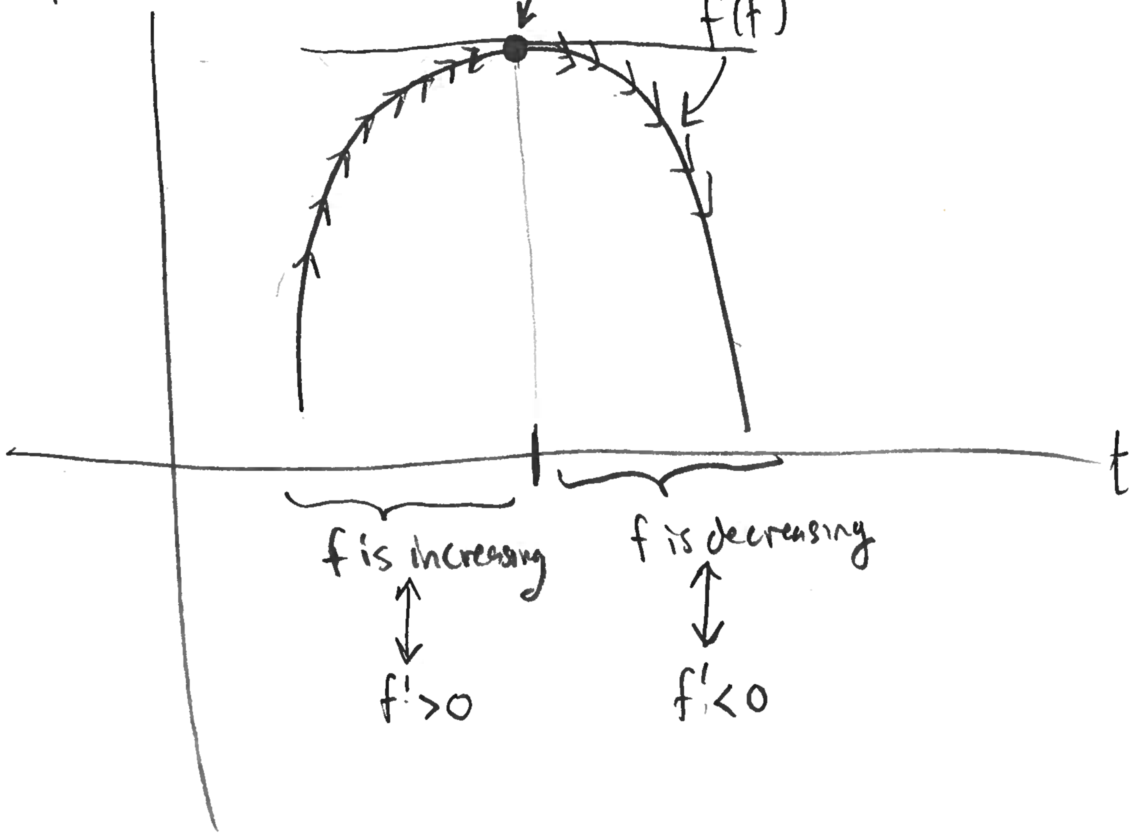
$f''(t)<0 \Longrightarrow$ f is concave down at t

Optimization – process of finding "relative" maximums or "relative" minimums

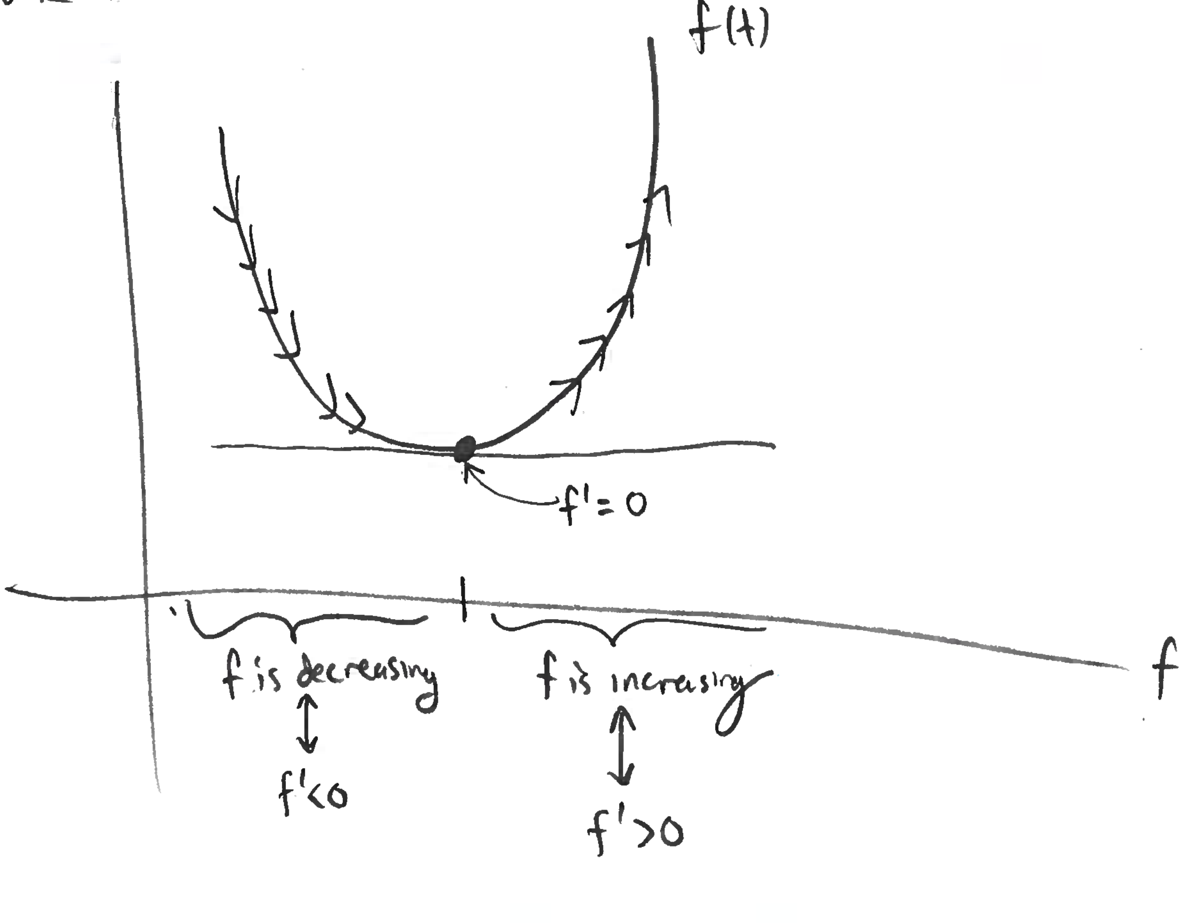
(prefer "local max" and "local min")



Look at a local max:



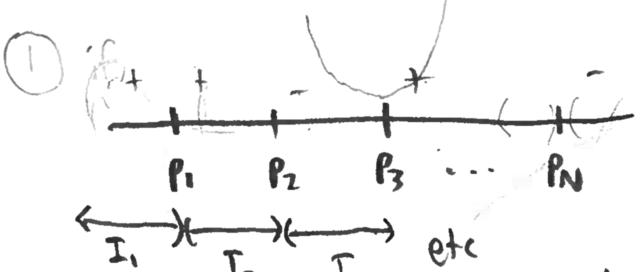
look at local min:



Def: A critical point is a place (value t) such that $f'(t)=0$

OR $f'(t)$ is undefined while $f(t)$ is defined.

The first derivative test: Let $P_1, P_2, \ldots, P_N$ be the crit pts; $P_1 \le P_2 \le \cdots \le P_N$.



Evaluate a test point from each interval $I_1, I_2, \ldots, I_{N+1}$ in the function f'! Note if each is $\oplus$, $\ominus$, or 0.

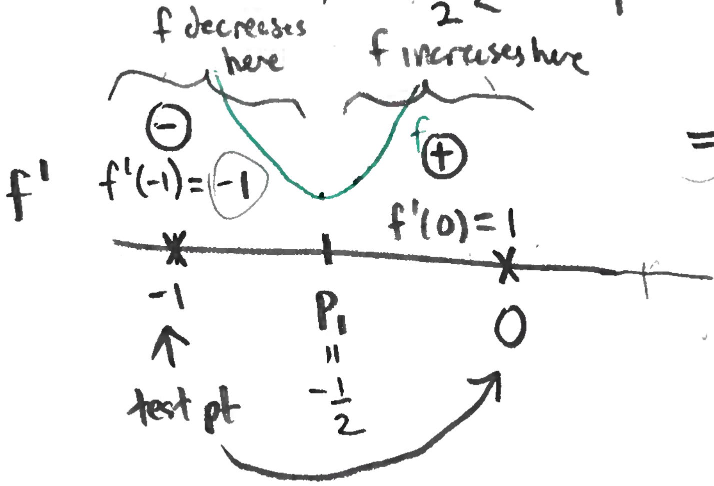
① We say f has a local min at P_i if the interval to its left has a $\ominus$ + the int to right has $\oplus$.

② We say f has a local max at P_i if ________ $\oplus$ ________ $\ominus$

Ex: Find all local extrema (min or max) of $f(x) = x^2 + x - 2$.

Soln: $f'(x) = \boxed{2x+1 \overset{\text{set}}{=} 0}$

$x = -\frac{1}{2}$ ← my only crit pt



$\Rightarrow$ f has a local min at $x = -\frac{1}{2}$

Ex: $f(x) = 2x^3 - 8x^2 + 6x - 1$

$$f'(x) = \boxed{6x^2 - 16x + 6 \overset{set}{=} 0}$$

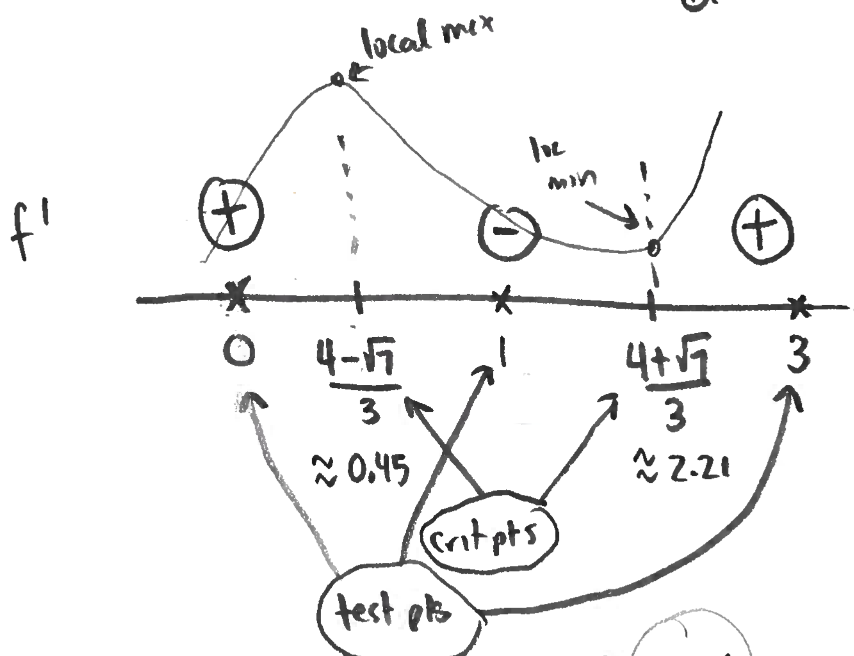
$$\cancel{2}(\underbrace{3x^2 - 8x + 3}) = 0$$

64 − 36

$\sqrt{28} = \sqrt{7 \cdot 4} = 2\sqrt{7}$

QF: $x = \dfrac{-(-8) \pm \sqrt{(-8)^2 - 4(3)(3)}}{2(3)}$

$$= \frac{8 \pm \sqrt{28}}{6} = \frac{4}{3} \pm \frac{\sqrt{7}}{3} \leftarrow \text{crit pts}$$



$f'(0) = 6 > 0$

$f'(1) = 6 - 16 + 6 < 0$

$f'(3) = 6 \cdot 9 - 16 \cdot 3 + 6$

$= \cancel{54}$

$60 - 16 \cdot 3 > 0$

$60 - 48$

$\begin{array}{r} 16 \\ \times 3 \\ \hline 48 \end{array}$

$\Rightarrow$ f has local max of 0.26 occurring at $x = \dfrac{4-\sqrt{7}}{3} \approx 0.45$

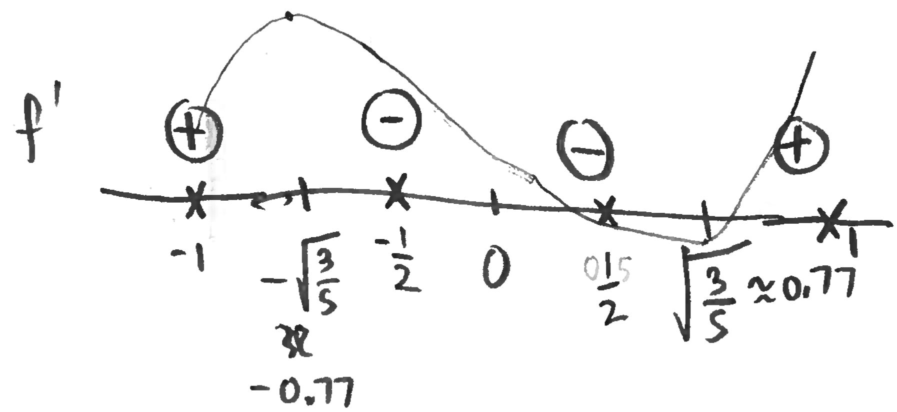
and f has local min of −5.23 occurring at $x = \dfrac{4+\sqrt{7}}{3} \approx 2.21$

Ex: $f(x) = 5x + \frac{3}{x} = 5x + 3x^{-1}$

$$f'(x) = \boxed{5 - \frac{3}{x^2} \overset{\text{set}}{=} 0}$$

$$5 = \frac{3}{x^2} \rightarrow \frac{1}{5} = \frac{x^2}{3} \rightarrow \frac{3}{5} = x^2 \rightarrow \boxed{x = \pm\sqrt{\frac{3}{5}}}$$

also f' undefined at $\boxed{x=0}$ ← crit pts



$f'(-1) =$

$f'(-\frac{1}{2}) =$

$f'(\frac{1}{2}) =$

$f'(1) =$

$\Rightarrow$ f has a local max at $x = -\sqrt{\frac{3}{5}}$

+ f has local min at $x = \sqrt{\frac{3}{5}}$