Ex: Length of a rectangle is increasing at a rate of $3\,\frac{cm}{s}$ & its width is increasing at a rate of $1\,\frac{cm}{s}$.

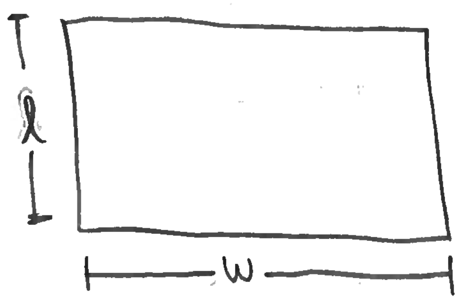
$\boxed{\frac{d\ell}{dt} = 3}$ $\boxed{\frac{dw}{dt} = 1}$

How fast is area increasing when length is 15cm & width is 12 cm?

"What is $\frac{dA}{dt}$?" $\boxed{\text{when } \ell = 15 \text{ and } w = 12}$

$A_{\square} = \ell \cdot w \text{ cm}^2$

Soln:



Given info

$\frac{d\ell}{dt} = 3$ $\frac{dw}{dt} = 1$

Goal: find $\left.\frac{dA}{dt}\right|_{\ell=15,\, w=12}$

plug in AFTER finding $\frac{dA}{dt}$

From $A = \ell \cdot w$

↓ take $\frac{d}{dt}$ product!

$$\frac{dA}{dt} = \frac{d}{dt}(\ell w) \overset{\text{prod}}{\underset{\text{Rule}}{=}} w \frac{d\ell}{dt} + \ell \frac{dw}{dt}$$

goal!

$$\overset{\text{given}}{=} 3w + 1\ell$$

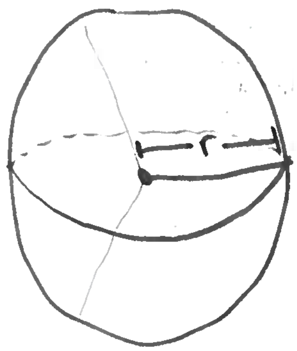
So, $\left.\frac{dA}{dt}\right|_{\ell=15,\, w=12} = 3(12) + 15 = 36 + 15 = 51\ \frac{cm^2}{s}$

Ex: If radius of a sphere is decreasing at a constant rate of $2\frac{cm}{s}$, then what rate is volume changing when the radius is 5cm?

$\frac{dr}{dt} = -2$

find $\frac{dVol}{dt}$

Soln:



Given

$\frac{dr}{dt} = -2 \ \frac{cm}{s}$

Goal:

r msr in cm

find $\left.\frac{dV}{dt}\right|_{r=5}$ when $V = \frac{4}{3}\pi r^3 \ cm^3$

$V = \frac{4}{3}\pi r^3$

$\downarrow$ take $\frac{d}{dt}$

$\frac{dV}{dt} = \frac{d}{dt}\frac{4}{3}\pi r^3 = \frac{4}{3}\pi \frac{d}{dt} r^3$

V goal!

a number

mismatch! $\rightarrow$ chain rule

$= \frac{4}{3}\pi \frac{dr}{dt}\frac{d}{dr} r^3$ $\leftarrow$ power rule

match!

$= \frac{4}{\cancel{3}}\pi \frac{dr}{dt}(\cancel{3}r^2)$

$= 4\pi r^2 \frac{dr}{dt}$

$= -8\pi r^2$

So, $\left.\frac{dV}{dt}\right|_{r=5} = -8\pi(5^2) = -200\pi \ \frac{cm^3}{s}$

EX: If two resistors w/ resistences R_1 and R_2 are connected in parallel, then total resistance R is given by

$$\frac{dR_1}{dt} = -0.1$$

$$(\star) \qquad \frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$$

If R_1 is decreasing at rate of $0.1\frac{\Omega}{s}$ and R_2 is increasing at rate of $0.2\frac{\Omega}{s}$, then how fast is R changing when $R_1 = 60\Omega$ and $R_2 = 90\Omega$?

$\frac{dR}{dt} = 0.2$

find $\frac{dR}{dt}$

Soln:

Given

$$\frac{dR_1}{dt} = -0.1 \qquad \frac{dR_2}{dt} = 0.2$$

Goal

$$\left.\frac{dR}{dt}\right|_{R_1=60,\, R_2=90}$$

So, take $\frac{d}{dt}$ of $(\star)$:

$$\frac{d}{dt}\left(\frac{1}{R}\right) = \frac{d}{dt}\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$

quot rule

$$\frac{R \cdot 0 - 1 \cdot \frac{dR}{dt}}{R^2} = \frac{R_1 \cdot 0 - 1 \cdot \frac{dR_1}{dt}}{R_1^2} + \frac{-\frac{dR_2}{dt}}{R_2^2}$$

$$-\frac{1}{R^2}\frac{dR}{dt} = -\frac{1}{R_1^2}\frac{dR_1}{dt} - \frac{1}{R_2^2}\frac{dR_2}{dt}$$

$$\frac{dR}{dt} = \frac{R^2}{R_1^2}\frac{dR_1}{dt} + \frac{R^2}{R_2^2}\frac{dR_2}{dt} = -0.1\frac{R^2}{R_1^2} + 0.2\frac{R^2}{R_2^2}$$

$$\frac{\Omega^2}{\Omega^2} \quad \frac{\Omega}{s} \quad \frac{\Omega^2}{\Omega^2} \quad \frac{\Omega}{s}$$

By (*), if $R_1=60$ and $R_2=90$,

$$\frac{1}{R}=\frac{1}{60}+\frac{1}{90}$$

$$R=\frac{1}{\frac{1}{60}+\frac{1}{90}}=36$$

So,

$$\left.\frac{dR}{dt}\right|_{R_1=60,R_2=90}=(-0.1)\frac{36^2}{60^2}+(0.2)\frac{36^2}{90^2}\approx -0.04\,\frac{\Omega}{s}$$

So resistance is decreasing at rate of $0.04\,\frac{\Omega}{s}$.