Ex: A bacterial colony is growing as a function

$$p(t) = \frac{200e^t}{1+e^t} \text{ bacteria at time } t$$

t ↑ days

What is growth rate on $\underbrace{3^{rd} \text{ day}}_{t=3}$?

rate ↑ derivative

logistic function

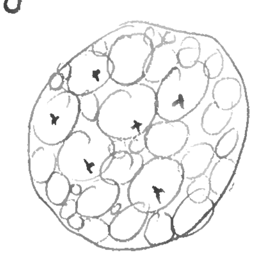
Soln: $$p'(t) = \frac{d}{dt}\left[\frac{200e^t}{1+e^t}\right]$$

quot Rule

$$= 200\left[\frac{(1+e^t)\frac{d}{dt}e^t - e^t\frac{d}{dt}[1+e^t]}{(1+e^t)^2}\right] \frac{\text{bacteria}}{\text{day}}$$

$$= 200\left[\frac{e^t + \cancel{e^{2t}} - \cancel{e^{2t}}}{(1+e^t)^2}\right] \frac{\text{bacteria}}{\text{day}}$$

$$= \frac{200e^t}{(1+e^t)^2} \frac{\text{bact}}{\text{day}}$$



So at day 3,

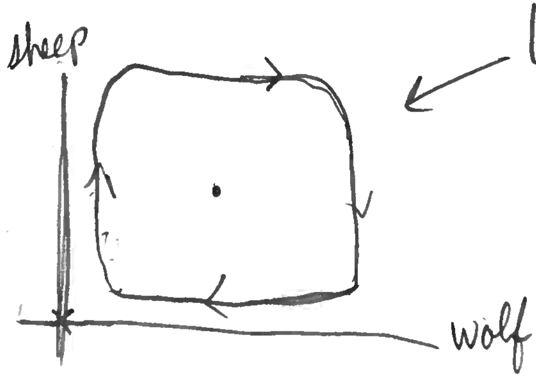
$$p'(3) = \frac{200e^3}{(1+e^3)^2} \approx 9 \frac{\text{bacteria}}{\text{day}}$$

Notice the growth rate decreases as time goes on:

$$p'(4) \approx 4 \frac{\text{bact}}{\text{day}}$$

$$p'(5) \approx 1 \frac{\text{bact}}{\text{day}}$$

Ex: Predator-prey model



Lotka-Volterra model

Obeys something like

a · #caribou

$$\begin{cases} \dfrac{dC}{dt} = aC - bCW \\ \dfrac{dW}{dt} = -cW + dCW \end{cases}$$

change in caribou pop

change in wolf pop

system of two differential equations

$-c \cdot$ #wolves

CW — interactions between wolves + caribou

(a) What values of $\frac{dC}{dt}$ and $\frac{dW}{dt}$ correspond to stable populations?

0 ~ means "no change"

(b) Spz $a=0.01$, $b=0.02$, $c=0.03$, $d=0.005$

stable pop pair comes from

$$\begin{cases} 0 = 0.01C - 0.02CW & \text{(i)} \\ 0 = -0.03C + 0.005CW & \text{(ii)} \end{cases}$$

~~Solve (i) for C:~~

From (i)

$$0 = C(0.01 - 0.02W)$$

$C = 0$

$\downarrow$ (ii)

$0 = 0$

useless — get a soln of form $(0, 0)$

both go extinct

$0.01 - 0.02W = 0$

$\downarrow$

$0.01 = 0.02W$

$$W = \frac{0.01}{0.02} = 0.5$$

$\downarrow$ (plug into (ii))

$$0 = -0.03C + 0.0025C$$

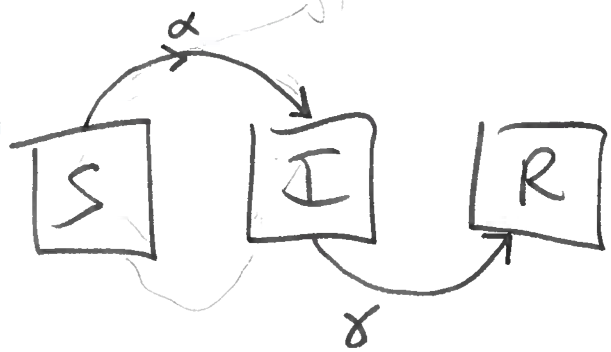
$C = 0$

seems like we always go extinct

SIR-model

Susceptible - Infected - Recovered

$$\begin{cases} \dfrac{dS}{dt} = -\alpha S I \\[2mm] \dfrac{dI}{dt} = \alpha S I - \gamma I \\[2mm] \dfrac{dR}{dt} = \gamma I \end{cases}$$

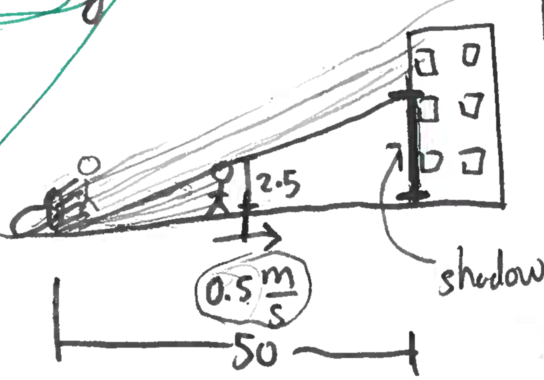


Ex: A spotlight on ground shining on wall 50m away. A women 2.5m tall walks from spotlight towards building at a speed of $0.5\,\frac{m}{s}$

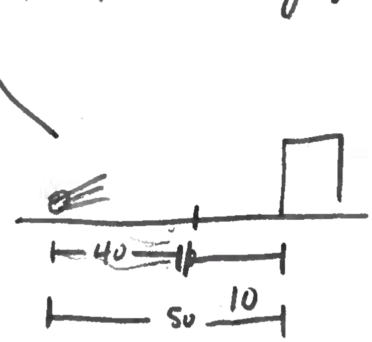
How fast is length of her shadow cast on building decreasing when she is 10 m from building?

derivative

Soln:

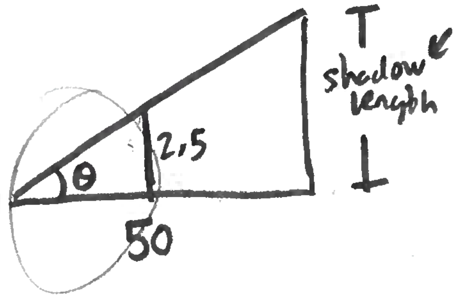


derivative of shadow length



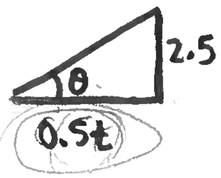
What is shadow length as a function of time?

$(0.5)(t) =$
↑
meters



$\longrightarrow \tan\theta = \frac{S_L}{50}$

equal!



$\longrightarrow \tan(\theta) = \frac{2.5}{0.5t}$

$$\frac{S_L}{50} = \frac{2.5}{0.5t}$$

$$\begin{array}{r} \overset{1}{125} \\ \times\ 2 \\ \hline 250 \end{array}$$

$$S_L = \frac{125}{0.5t} = \frac{250}{t} = 250t^{-1}$$

$$\boxed{\frac{dS_L}{dt} = \frac{-250}{t^2}}$$

At what time is she 10m from bldg?

$$0.5t = 40$$

$$\boxed{t = 80}$$

Therefore when she is 10 m from building, her shadow length is changing at rate

$$\frac{dS_L}{dt} = \frac{-250}{80^2} \approx -\frac{0.04\,m}{s}$$

So decreasing at rate of $\frac{0.04\,m}{s}$.