Recall : $\frac{d}{dx}\left[x^{\alpha}\right] = \alpha x^{\alpha-1}$ ← Power Rule

Ex : Calculate $f'(2)$ if

$$f(x) = 4x^{-5} + 9x^{-1}$$

Soln :

$$f'(x) = \frac{d}{dx}\left[4x^{-5} + 9x^{-1}\right]$$

$$\overset{SR, CMR}{=} 4\frac{d}{dx}\left[x^{-5}\right] + 9\frac{d}{dx}\left[x^{-1}\right]$$

$$\overset{PR}{=} 4(-5)x^{-6} + 9(-1)x^{-2}$$

$$= -20x^{-6} - 9x^{-2}$$

So,

$$f'(2) = -20(2^{-6}) - 9(2^{-2}) \approx -2.56$$

Ex : Find eqt of tangent line to $g(w) = (\sqrt[3]{w})w$ at $w = 2$.

Soln : $g(2) = (\sqrt[3]{2})(2) = 2\sqrt[3]{2} \rightsquigarrow \boxed{(2, 2\sqrt[3]{2})}$ on tan line

$$g(w) = (\sqrt[3]{w})w = (w^{1/3})w^{1} = w^{1/3+1} = w^{4/3}$$

$$y - y_0 = m(x - x_0)$$

So, $g'(w) = \frac{d}{dw}\left[w^{4/3}\right] = \frac{4}{3}w^{\frac{4}{3}-1} = \frac{4}{3}w^{1/3}$

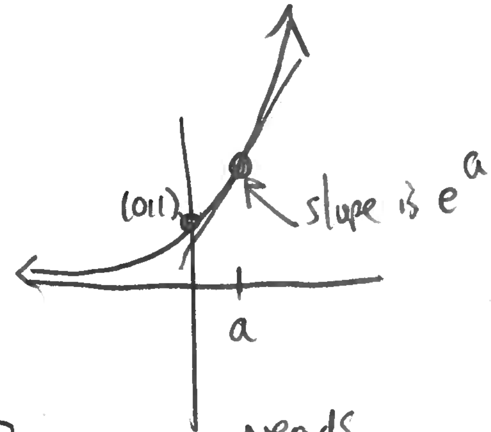
So, $\boxed{g'(2) = \frac{4}{3}(2^{1/3})}$ ← slope of tan line

$$y - 2\sqrt[3]{2} = \frac{4}{3}2^{1/3}(x - 2)$$

Other nonpolynomial functions

e^x ~ very special!

"eigenfunction of $\frac{d}{dx}$"



Why is it not $\frac{d}{dx}[e^x] = xe^{x-1}$ → meaning $\boxed{\frac{d}{dx}[e^x] = e^x}$

$\sin(x)$ ~ $\boxed{\frac{d}{dx}[\sin(x)] = \cos(x)}$ ← Needs $\boxed{\lim_{x\to 0} \frac{\sin x}{x} = 1}$

↑

RADIANS

(written HW related to this)

↓

$\cos(x)$ ~ $\boxed{\frac{d}{dx}[\cos(x)] = -\sin(x)}$ ← NEED: $\boxed{\lim_{x\to 0} \frac{\cos(x)-1}{x} = 0}$

Why:

$$\frac{d}{dx}\cos(x) = \lim_{h\to 0} \frac{\cos(x+h)-\cos(x)}{h}$$

$$= \lim_{h\to 0} \frac{\cos(x)\cos(h) - \sin(x)\sin(h) - \cos(x)}{h}$$

$$= \lim_{h\to 0} \frac{\cos(x)[\cos(h)-1]}{h} - \sin(x)\frac{\sin(h)}{h}$$

$$= \cos(x)\underbrace{\left[\lim_{h\to 0} \frac{\cos(h)-1}{h}\right]}_{=0} - \sin(x)\underbrace{\left[\lim_{h\to 0}\frac{\sin(h)}{h}\right]}_{=1}$$

$$= -\sin(x)$$

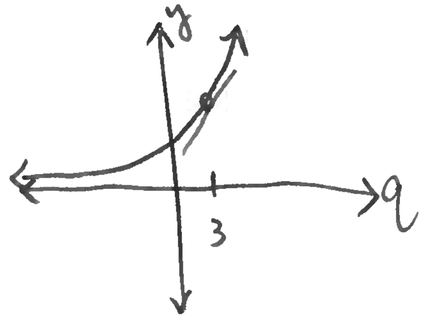
Written HW

Show $\frac{d}{dx}\sin x = \cos x$ using limit like we did here

NEED: $\sin(x+h) = \sin(x)\cos(h) + \sin(h)\cos(x)$

Ex: Find tan line to $h(q)=e^q$ at $q=3$.

Soln:



$h(3)=e^3 \longrightarrow (3,e^3)$ on graph

$h'(q)=\frac{d}{dq}[e^q]=e^q$

$h'(3)=e^3 \leftarrow$ slope

$$y-e^3=e^3(q-3)$$

Ex: Find eqt for tan line of $A(\alpha)=\sin(\alpha)+\cos(\alpha)$ at $\alpha=\frac{\pi}{4}$.

Soln: $A(\frac{\pi}{4})=\sin(\pi/4)+\cos(\pi/4)=\frac{\sqrt{2}}{2}+\frac{\sqrt{2}}{2}=\sqrt{2}$

$\Downarrow$

$(\frac{\pi}{4},\sqrt{2})$ on graph

$A'(\alpha)=\frac{d}{d\alpha}[\sin(\alpha)+\cos(\alpha)] \underset{SR}{=} \frac{d}{d\alpha}[\sin\alpha]+\frac{d}{d\alpha}[\cos(\alpha)]$

$= \cos(\alpha)-\sin(\alpha)$

So,

$A'(\frac{\pi}{4})=\cos(\frac{\pi}{4})-\sin(\frac{\pi}{4})=\frac{\sqrt{2}}{2}-\frac{\sqrt{2}}{2}=0$

So, tan line is

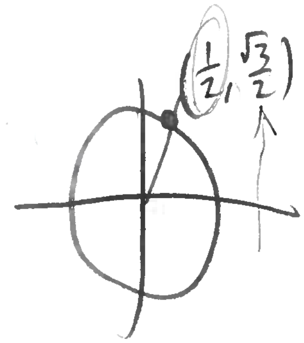
$y-\sqrt{2}=\cancel{0(\alpha-\frac{\pi}{4})}$

$y=\sqrt{2}$

Ex: Find eqt of tan line to

$$B(\gamma) = e^{\gamma} + \cos(\gamma)$$

at $\gamma = \frac{\pi}{3}$.



Soln: $B\left(\frac{\pi}{3}\right) = e^{\pi/3} + \cos(\pi/3) = e^{\pi/3} + \frac{1}{2}$

$$\Downarrow$$

$$\left(\frac{\pi}{3}, e^{\pi/3} + \frac{1}{2}\right) \text{ on graph}$$

$$B'(\gamma) = \frac{d}{d\gamma}\left[e^{\gamma} + \cos(\gamma)\right] = e^{\gamma} - \sin(\gamma)$$

$$B'\left(\frac{\pi}{3}\right) = e^{\pi/3} - \sin(\pi/3) = e^{\pi/3} - \frac{\sqrt{3}}{2}$$

$$\Downarrow$$

Tan line is

$$y - \left(e^{\pi/3} + \frac{1}{2}\right) = \left(e^{\pi/3} - \frac{\sqrt{3}}{2}\right)\left(\gamma - \frac{\pi}{3}\right)$$