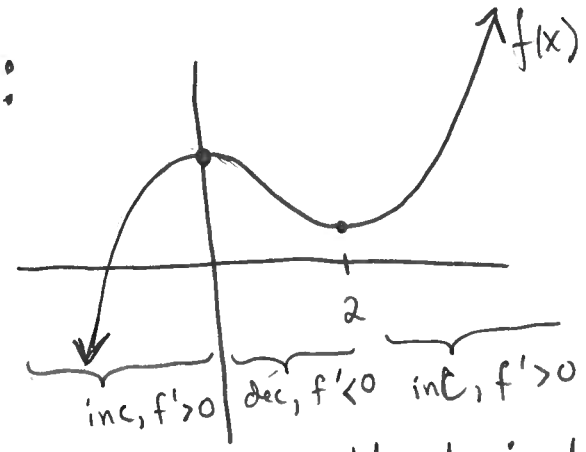
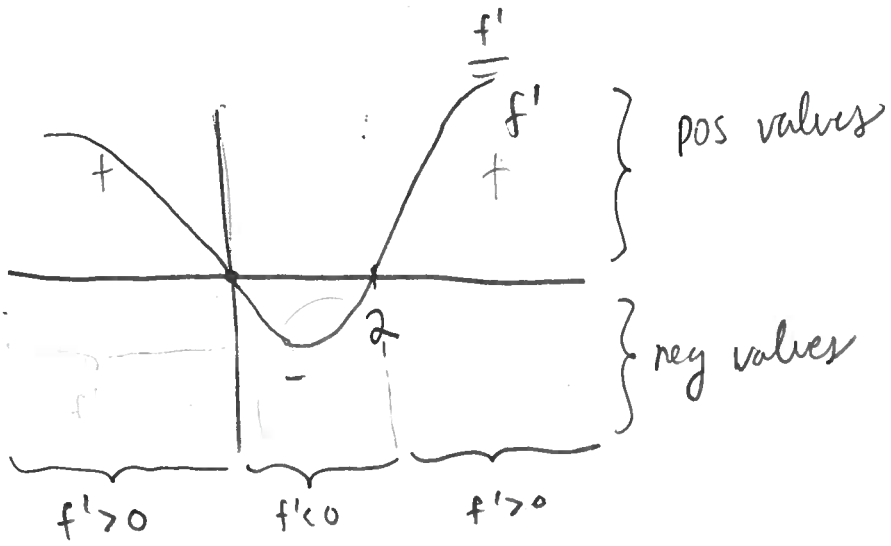
increasing $\longleftrightarrow f' > 0$ decreasing $\longleftrightarrow f' < 0$

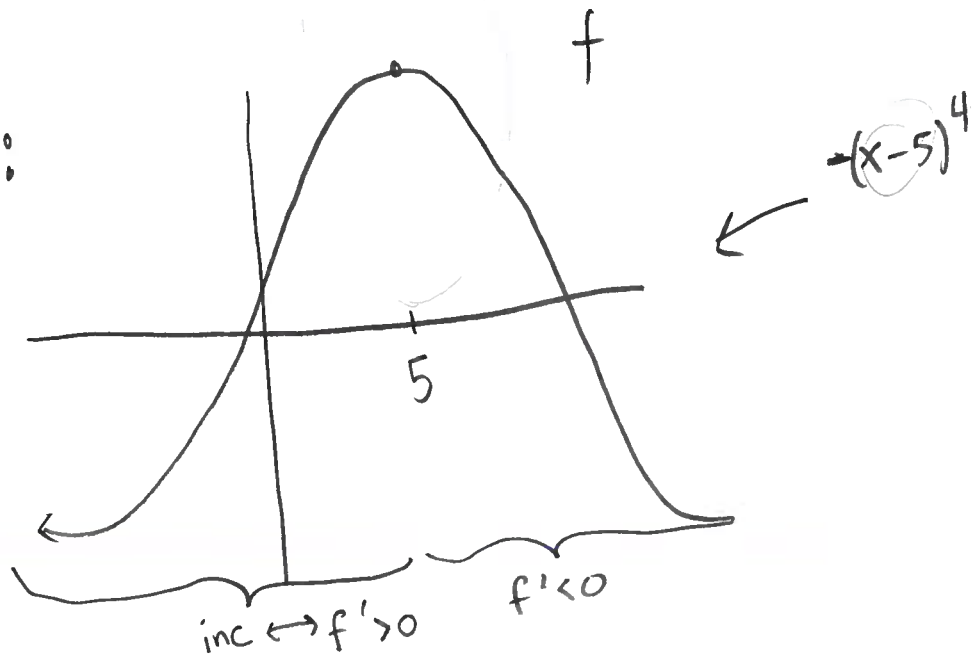
EX:

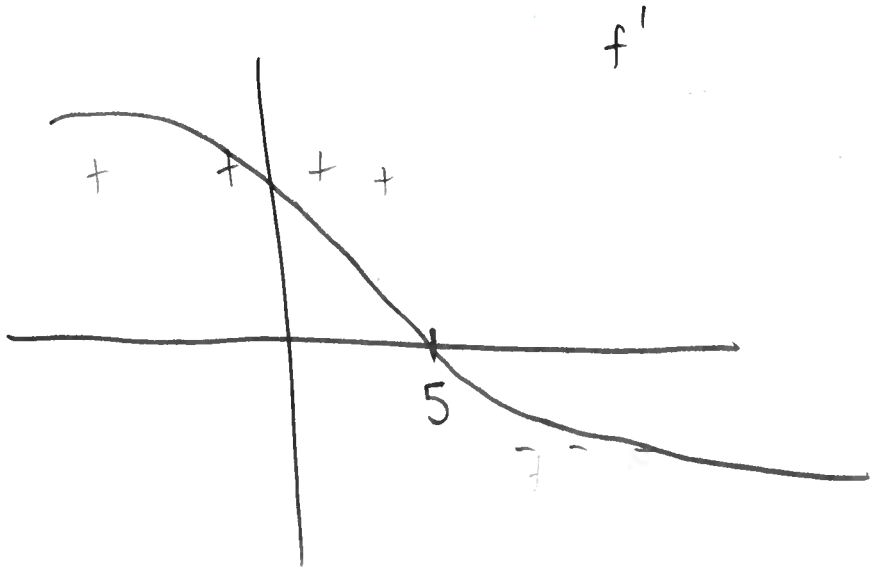


Goal: sketch the derivative

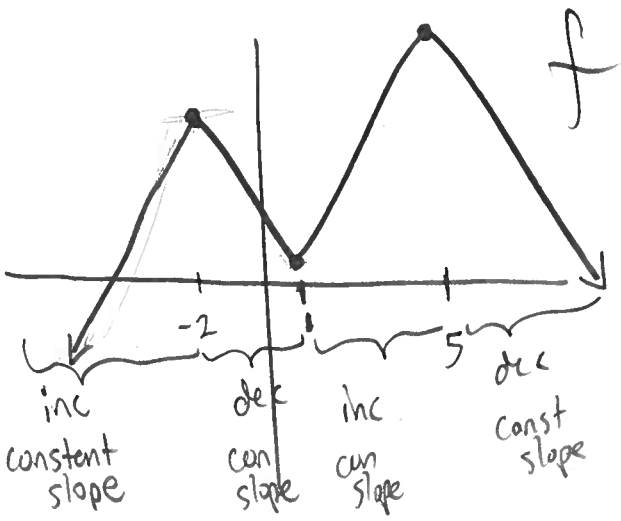


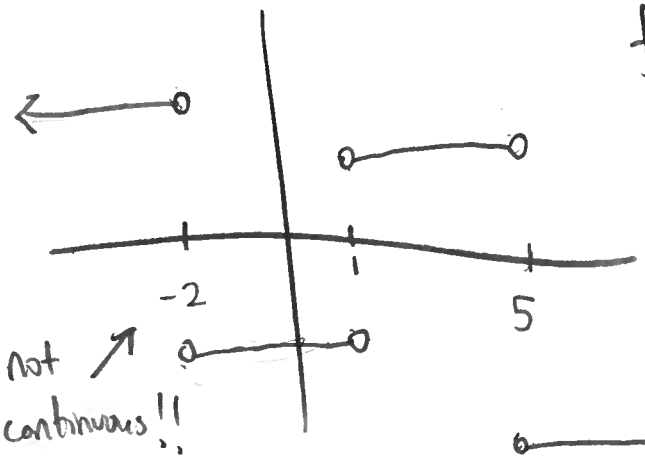
EX:



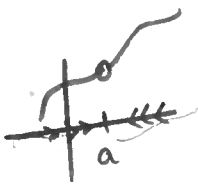


Ex:





$\lim_{x \to a} f(x)$

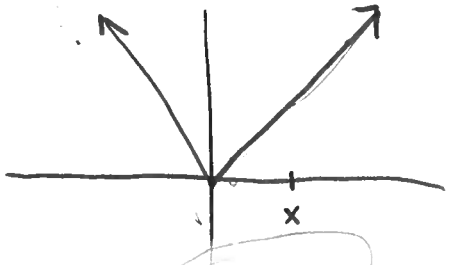


$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

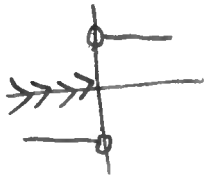
$$\frac{d}{dx} x^n = n x^{n-1}$$

$x = x^1 \quad \frac{d}{dx} x^1 = 1 x^{1-1} = 1 x^0 = 1$

Ex: Find $f'(x)$ if $f(x) = |x|$



$$|x| = \begin{cases} x, & x > 0 \\ 0, & x = 0 \\ -x, & x < 0 \end{cases}$$



$x > 0$

$$f'(x) = \frac{d}{dx} x = 1$$

$x < 0$

$$f'(x) = \frac{d}{dx}(-x) = -\frac{d}{dx}(x) = -1$$

no derivative exists at $x = 0$

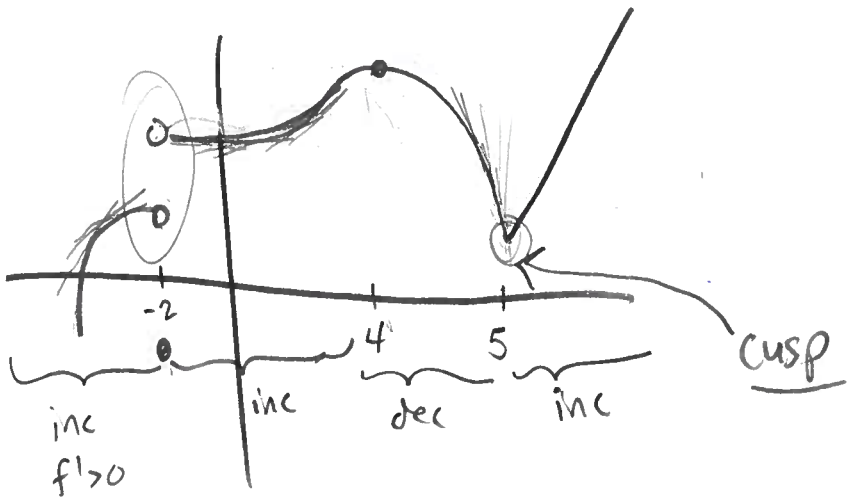
$x = 0$

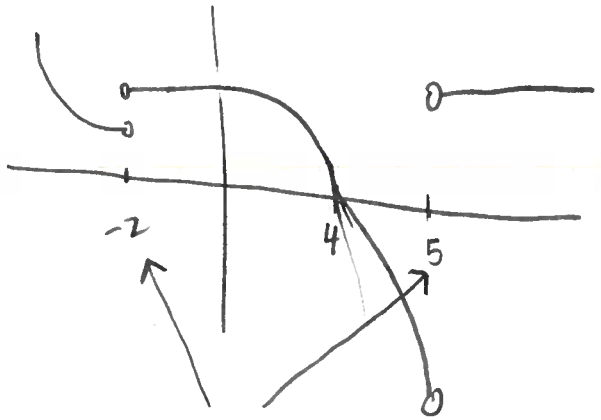
$$\lim_{h \to 0^-} \frac{f(0+h) - f(0)}{h} = \lim_{h \to 0^-} \frac{-h - 0}{h} = -1$$

($h < 0$)

$$\lim_{h \to 0^+} \frac{f(0+h) - f(0)}{h} = \lim_{h \to 0^+} \frac{h - 0}{h} = 1$$

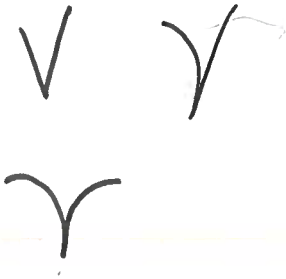
EX: Indicate where (x-values) f is not diff'bl.



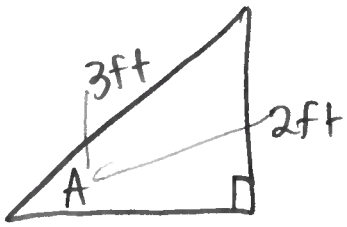


x=-2, 5

not diff'bl



Units - behave like variables



~~tan(A)~~

$\sin(A) = \frac{2\text{ft}}{3\text{ft}} = \frac{2}{3}$ ← pure unitless quantity

1min = 60 sec

$\frac{60\text{sec}}{1\text{min}} = 1$

How many seconds in 5min?

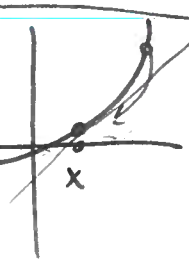
$$5\text{min} = (5\text{min})(1) = (5\text{min})\left(\frac{60\text{sec}}{1\text{min}}\right)$$

$$= 300\text{sec}$$

$$f'(x) \overset{\text{def}}{=} \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$f(x+h) - f(x)$ ← has units of f

h ← has units of the indep var



Ex: $f(t)$ is measured in miles

t is measured in hours

⇓

$f'(t)$ is measured in $\frac{\text{mi}}{\text{hr}}$ ← miles (per) hour ← divide

velocity

$g(t) = f'(t)$ ← is measured in $\frac{\text{mi}}{\text{hr}}$

$f''(t) = g'(t)$ ← measured in $\frac{\left(\frac{\text{mi}}{\text{hr}}\right)}{\frac{\text{hr}}{1}} = \frac{\text{mi}}{\text{hr}^2}$ acceleration

↑ called 2nd derivative of f

$f(t)$ miles

t hours

function	units	name
$f(t)$	miles	distance
$f'(t)$	$\frac{\text{miles}}{\text{hr}}$	velocity ✓
$f''(t)$	$\frac{\text{mi}}{\text{hr}^2}$	acceleration
$f'''(t)$	$\frac{\text{mi}}{\text{hr}^3}$	jerk
$f^{(4)}(t) = f''''(t)$	$\frac{\text{mi}}{\text{hr}^4}$	snap
$f^{(5)}$	$\frac{\text{mi}}{\text{hr}^5}$	crackle
$f^{(6)}$	$\frac{\text{mi}}{\text{hr}^6}$	pop

Ex: $P(A)$ profits \$

A sales quantity of books

$P'(A)$ $\frac{\$}{\text{book}}$ $P''(s)$ $\frac{\$}{\text{book}^2}$ etc...