$f'(a) = [\text{slope of } f(x) \text{ at } x=a] \overset{\text{def}}{=} \lim_{h\to 0} \frac{f(a+h)-f(a)}{h}$

"f prime of a"

Ex: Find $f'(x)$ for $f(x) = \frac{1}{x}$

Soln: Calculate

$$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$$

$$= \lim_{h\to 0} \frac{\frac{1}{x+h} - \frac{1}{x}}{h}$$

$$= \lim_{h\to 0} \frac{1}{h}\left[\frac{\cancel{x} - (\cancel{x}+h)}{x(x+h)}\right]$$

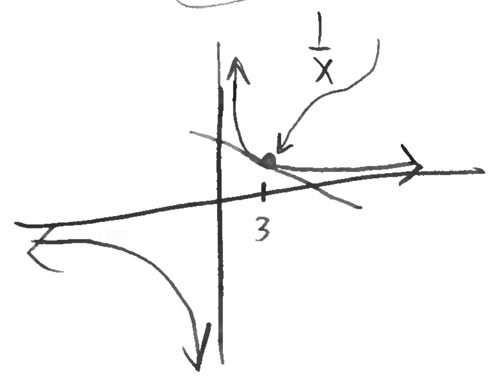
$$= \lim_{h\to 0} \frac{1}{\cancel{h}}\left[\frac{-\cancel{h}}{x(x+h)}\right]$$

$$= - \boxed{\lim_{h\to 0} \frac{1}{x(x+h)}}$$

$$\rightarrow \; = -\frac{1}{x^2}$$

$$\frac{1}{x+h} = \frac{x}{x(x+h)}$$

$$\frac{1}{x} = \frac{x+h}{x(x+h)}$$



Use this to find eqt for tan line of $f(x) = \frac{1}{x}$ at $x=3$:

point: $(3, f(3)) = (3, \frac{1}{3})$ (x_0, y_0)

slope: $f'(3) = -\frac{1}{3^2} = -\frac{1}{9}$

$$y - \frac{1}{3} = -\frac{1}{9}(x-3) \longrightarrow y = -\frac{x}{9} + \frac{3}{9} + \frac{3}{9}$$

$$= -\frac{x}{9} + \frac{6}{9}$$

Ex: Let $f(x) = 3x^2 + x + 1$

~~Sol~~: $f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$

$= \lim_{h\to 0} \frac{[3(x+h)^2 + (x+h) + 1] - [3x^2 + x + 1]}{h}$

$(x+h)^2 = \boxed{x^2 + 2xh + h^2}$

$= \lim_{h\to 0} \frac{\cancel{3x^2} + 6xh + 3h^2 + \cancel{x} + h + \cancel{1} - \cancel{3x^2} - \cancel{x} - \cancel{1}}{h}$

$= \lim_{h\to 0} \frac{\cancel{h}(6x + 3h + 1)}{\cancel{h}}$

$= \lim_{h\to 0} 6x + \cancel{3h} + 1$

$\longrightarrow = 6x + 1$

Eqt of tan line at $x = -3$:

$f(-3) = 3(-3)^2 + (-3) + 1$
$= 27 - 3 + 1$
$= 25$

point: $(-3, f(-3)) = (-3, 25)$
slope: $f'(-3) = 6(-3) + 1 = -18 + 1 = -17$

$y - 25 = -17(x - (-3))$

$\begin{array}{r} {}^{2}17 \\ \times\ 3 \\ \hline 51 \end{array}$

$y = -17x \underbrace{- 17(3) + 25}_{-51+25}$

$= -17x - 26$

Ex: The limit

$$\lim_{\Delta x \to 0} \frac{3(x+\Delta x)^3+1 - (3x^3+1)}{\Delta x}$$

same as h

represents $f'(x)$ for some function f.

What is f?

$$\boxed{f(x) = 3x^3+1}$$

Ex: This

$$\lim_{\xi \to 0} \frac{(2+\xi)^5 - 2^5}{\xi}$$

represents $f'(a)$ for some f and some a.

$f(x) = x^5$

$a = 2$

Alternate notations:

$$f'(a) = \underbrace{\left.\frac{df}{dx}\right|_{x=a}} = Df(a) = \dot{f}(a) = \frac{d}{dx}f$$

"Compute deriv, then plug in x=a"

physics

2nd 1st 3rd 5th

Ex: Find $f'(x)$ when $f(x) = 8$

$$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$$

$$= \lim_{h\to 0} \frac{8-8}{h} = \lim_{h\to 0} 0 = 0$$

Similarly: if C constant and $f(x)=C$, then $f'(x)=0$ "the derivative of f"

"the derivative of a constant is zero"

Find tan line at $(5,8)$.

$$y-8 = \cancel{0(x-5)}$$

$$\boxed{y=8}$$

Yesterday: we saw $f(x) = x^2 \longrightarrow f'(x) = 2x$

today: we saw $f(x) = \frac{1}{x} \longrightarrow f'(x) = -\frac{1}{x^2}$

Ex: $f(x) = x^3$, then

$$f'(x) = \lim_{h\to 0} \frac{(x+h)^3 - x^3}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{h}\left[x^2 + 2x^2 + 2xh + xh + h^2\right]}{\cancel{h}}$$

$$= \lim_{h\to 0} 3x^2 + \cancel{3xh} + \cancel{h^2}$$

$$= 3x^2$$

$$(x+h)^3 = (x+h)^2(x+h)$$
$$= (x^2 + 2xh + h^2)(x+h)$$
$$= x^2(x+h) + 2xh(x+h) + h^2(x+h)$$
$$= \cancel{x^3} + x^2h + 2x^2h + 2xh^2 + xh^2 + h^3$$

In general: if $f(x) = x^n$, $n = \pm 1, \pm 2, \pm 3, \dots$

then $\boxed{f'(x) = nx^{n-1}}$

in terms of "diff'l operator"

$$\boxed{\frac{d}{dx} x^n = nx^{n-1}, \quad n \in \{\pm 1, \pm 2, \dots\}}$$

$\underline{n=2}$

$$\frac{d}{dx} x^2 = 2x$$

$n=3 \longrightarrow \frac{d}{dx} x^3 = 3x^2$

$n=-1 \longrightarrow \frac{d}{dx} x^{-1} = \frac{d}{dx} \frac{1}{x} = -x^{-2}$

This will allow us to do a lot of things

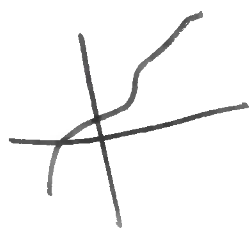
Evaluate $\lim_{h \to 0} \frac{(x+h)^{800} - x^{800}}{h}$

understand this is asking for $\frac{d}{dx} x^{800} = 800x^{799}$

prev page

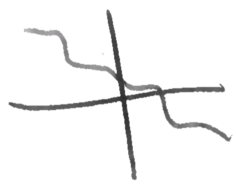
Connection between derivative + increasing/decreasing

f increasing:

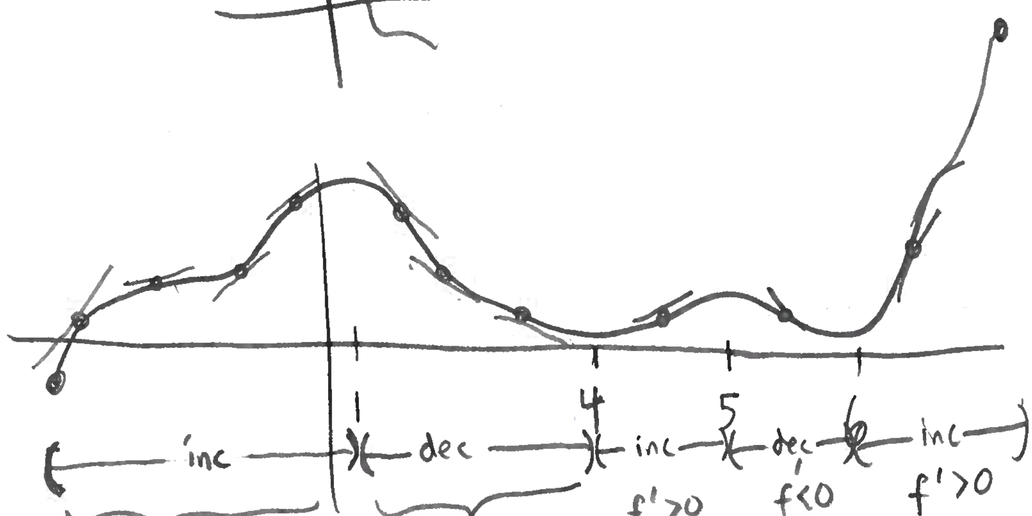


left-to-right it moves up

f decreasing:



left-to-right it moves down



f'>0 b/c all slopes are ⊕ slope

f'<0 b/c all slopes are ⊖

Rules of derivatives

$$\frac{d}{dx}\left[f(x)+g(x)\right] = \frac{df}{dx} + \frac{dg}{dx}$$

$$\frac{d}{dx}\left[c f(x)\right] = c\frac{df}{dx}$$

$$\cancel{\frac{d}{dx} f(x)g(x) \overset{?}{=} \left[\frac{d}{dx}f(x)\right]\left[\frac{dg}{dx}\right]} \; ?$$

NO

$$\frac{d}{dx}(x^2) \overset{?}{=} \left(\frac{d}{dx}x\right)\left(\frac{d}{dx}x\right) = 1(1) = 1$$

$$\frac{d}{dx}(x^2) = 2x$$

not SAME