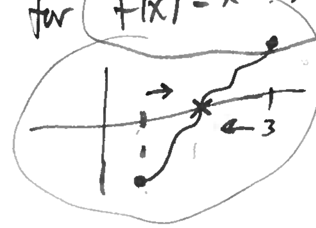
Ex: Use IVT to show there is a soln to equation $x^2+x-6=0$ in $[1,3]$.

$(x+3)(x-2)=0$

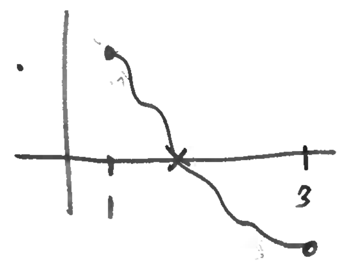
$x=2,-3$

Soln: It suffices to show that for $f(x)=x^2+x-6$, that ($f(1)<0$ and $f(3)>0$)



OR

($f(1)>0$ and $f(3)<0$).



$f(1)=1^2+1-6=-4$

$f(3)=3^2+3-6=9+3-6=6$

Using $\alpha=0$: $f(1)<\alpha=0<f(3)$

By IVT, since f is continuous (it is a polynomial) we can conclude that there is some c in $[1,3]$ such that $f(c)=0$.

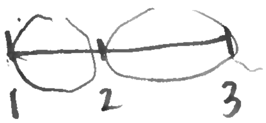
Million Dollar Problems

Navier Stokes equation

show a soln exists and is the only soln

We know there is a root of $f(x) = x^2 + x - 6$ in $[1,3]$. That means there is a root in either

$[1,2]$ or in $[2,3]$



$f(1) = -4$

$\boxed{f(2) = 2^2 + 2 - 6 = 0}$

we found it

$f(2) =$

$f(3) =$

Ex: Try to approximate a root of $f(x) = xe^x - 1$

Soln: $xe^x - 1 = 0$

$\cancel{xe^x = 1 \;/\; x = 1 \text{ or } e^x = 1}$

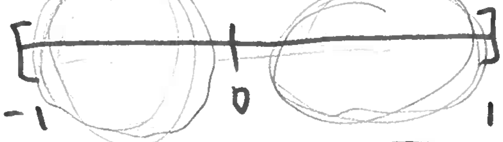
nothing in college algebra can solve this simple problem $\Longrightarrow$ try IVT

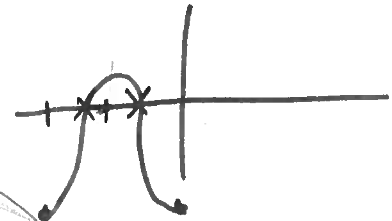
$f(-1) = -e^{-1} - 1 = -\frac{1}{e} - 1 < 0$

$f(1) = (1)e^1 - 1 = e - 1 > 0 \quad (e \approx 2.71)$

"1st iteration"

By IVT: there is a root of $f(x) = xe^x - 1$ in interval $[-1, 1]$.





2nd iteration →

$[-1, 0]$

$f(-1) = -\frac{1}{e} - 1$

$f(0) = 0e^0 - 1$

$= -1$

WOULD NEED

$\underbrace{-\frac{1}{e} < \alpha = 0 < -1}$

impossible!

Cannot make any conclusion about a root

$[0, 1]$

$f(0) = -1 < 0$

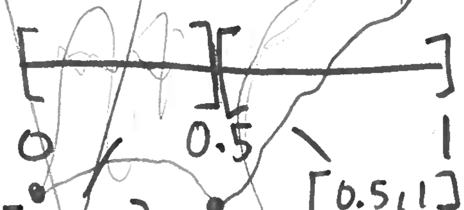
$f(1) = e - 1 > 0$

$-1 < \alpha = 0 < e - 1$ ✓

$\Downarrow$

Apply IVT to conclude a root is here!

3rd iteration: →



$[0, 0.5]$

$f(0) = -1$

$f(0.5) = 0.5e^{0.5} - 1$

$= 0.5\sqrt{e} - 1$

$= -0.175$

$[0.5, 1]$

$f(0.5) = -0.175$

$f(1) = e - 1 > 0$

the root must live in $[0.5, 1]$

$f(x) = xe^{x} - 1$

$\frac{1+0.5}{2}$

4th iteration:

0.5 0.75 1

$[0.5, 0.75]$ $[0.75, 1]$

$f(0.5) = -0.175$ $f(0.75) = 0.587$

$f(0.75) = 0.587$ $f(1) = e - 1$

5th iteration:

0.5 0.625 0.75

$f(0.5) = -0.175$ $f(0.625) = 0.167$

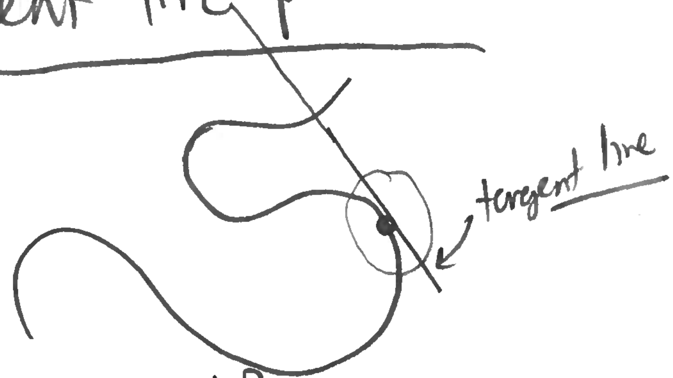
$f(0.625) = 0.167$ $f(0.75) = 0.587$

$\downarrow$

root lies in

$[0.5, 0.625]$

Tangent line problem



A tangent line at P is a straight line that touches curve at exactly one point @ nearby P.

"instanteneous slope"