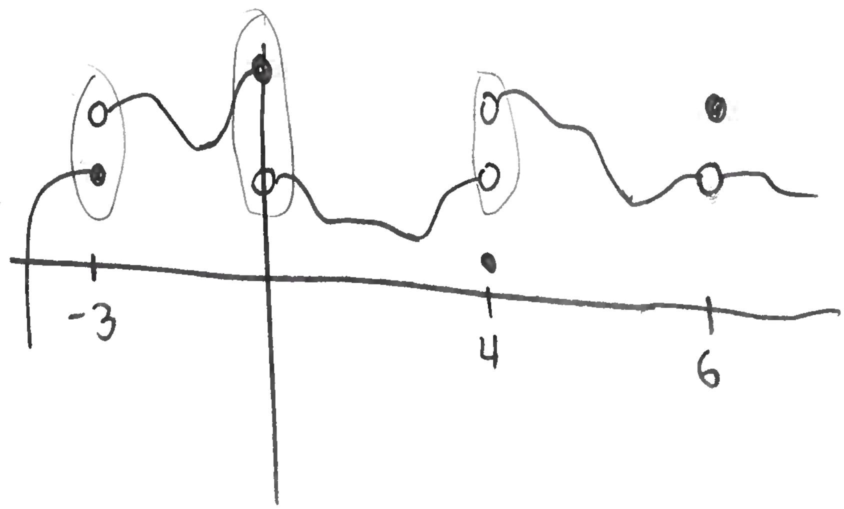
Ex: $\lim_{t \to -0.5^-} \sqrt{\frac{t-1}{t-7}} \underset{t=-0.5}{=} \sqrt{\frac{-0.5-1}{-0.5-7}} = \sqrt{\frac{-1.5}{-7.5}}$

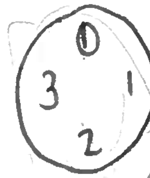
$= \sqrt{\frac{1.5}{7.5}}$

$\sqrt{8} = 2\sqrt{2}$

Ex: Indicate x-values for which f is not continuous ("discontinuous").



$x = -3, 0, 4, 6$



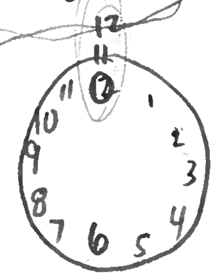
Ex: $f(x) = \dfrac{x^2+x-1}{(x-3)(x-10)}$

Use interval notation to indicate where f is continuous.

$a \cdot b = 0$

$\swarrow \quad \searrow$

$a=0 \quad b=0$



Soln: numerator: no issue

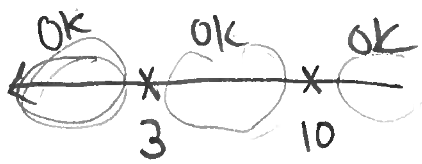
denominator: problems whenever

$(x-3)(x-10)=0$

$\swarrow \quad \searrow$

$x-3=0$ or $x-10=0$

$x=3 \qquad x=10$ ← problems



$(-\infty,3) \cup (3,10) \cup (10,\infty)$

~~3]~~ 3[inf I

EX: $$f(x) = \begin{cases} 8-x \; ; & x \leq -6 \\ 2+x \; ; & x > -6 \end{cases}$$

-6

$$\lim_{x \to -6^-} f(x) = \lim_{x \to -6^-} 8-x = 8-(-6) = 14$$

$$\lim_{x \to -6^+} f(x) = \lim_{x \to -6^+} 2+x = -4$$

$$\boxed{\lim_{x \to -6} f(x)} = \underline{DNE}$$

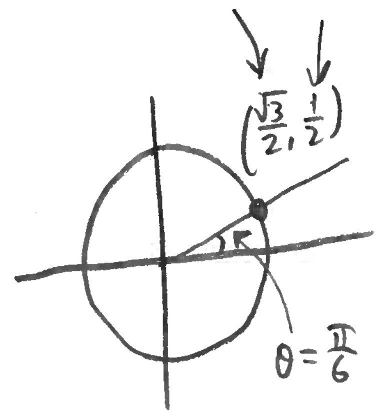
$$f(-6) = 8-(-6) = 14$$

is f continuous at -6?: $\underline{\text{NO}}$

Ex: Is

$\frac{\pi}{6}$

$$f(x) = \begin{cases} \sin(x) & ; \ x < \frac{\pi}{6} \\ \cos(x) & ; \ x \geq \frac{\pi}{6} \end{cases}$$



Continuous on $\mathbb{R}$ $\left((-\infty, \infty)\right)$?

Soln: $\lim\limits_{x \to \frac{\pi}{6}^-} f(x) = \lim\limits_{x \to \frac{\pi}{6}^-} \sin(x) = \sin\left(\frac{\pi}{6}\right) = \frac{1}{2}$

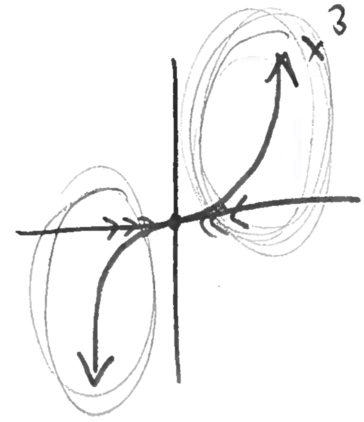
$\lim\limits_{x \to \frac{\pi}{6}^+} f(x) = \lim\limits_{x \to \frac{\pi}{6}^+} \cos(x) = \cos\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2}$

$\lim\limits_{x \to \frac{\pi}{6}} f(x)$ DNE

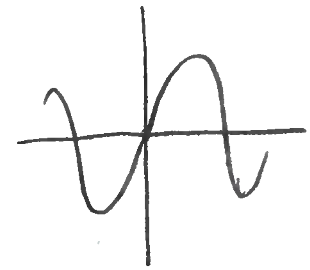
$\rightarrow$ no, it is not continuous at $x = \frac{\pi}{6}$

Ex (back to squeeze)

$$\lim_{x\to 0} x^3\cos\left(e^{\frac{1}{x}}\right)$$



$$-1 \le \cos\left(e^{\frac{1}{x}}\right) \le 1$$



$x<0$ (near 0)

mult by x^3

$$-x^3 \ge x^3\cos\left(e^{\frac{1}{x}}\right) \ge x^3$$

$$x^3 \le x^3\cos\left(e^{\frac{1}{x}}\right) \le -x^3$$

$$\lim_{x\to 0^-} x^3 = 0$$

$$\lim_{x\to 0^-} (-x^3) = 0$$

$\therefore$ by squeeze $\boxed{\lim_{x\to 0^-} x^3\cos\left(e^{\frac{1}{x}}\right) = 0}$

$x>0$ (near 0)

mult by x^3

$$-x^3 \le x^3\cos\left(e^{\frac{1}{x}}\right) \le x^3$$

$$\lim_{x\to 0^+} x^3 = 0$$

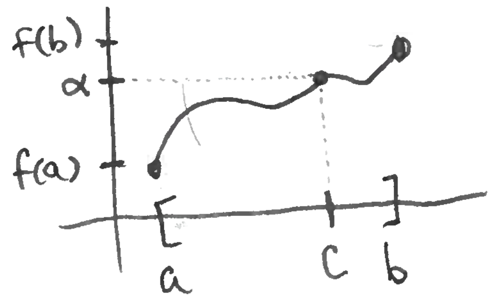
$$\lim_{x\to 0^+} (-x^3) = 0$$

$$\boxed{\lim_{x\to 0^+} x^3\cos\left(e^{\frac{1}{x}}\right) = 0}$$

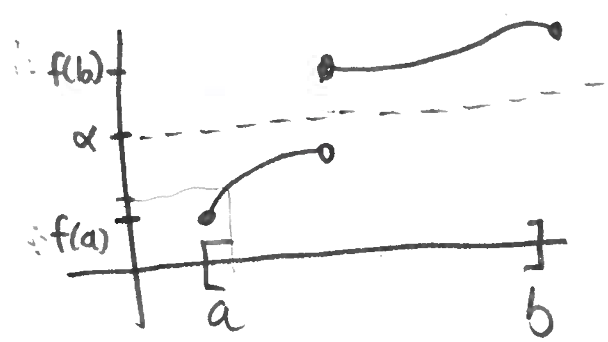
$$\lim_{x\to 0} x^3\cos\left(e^{\frac{1}{x}}\right) = 0$$

Intermediate value theorem

If f is continuous on an interval $[a, b]$ and if α is a number between $f(a)$ and $f(b)$ $\left(f(a) \le \alpha \le f(b) \text{ or } f(b) \le \alpha \le f(a)\right)$ then there is a number c in $[a,b]$ such that $f(c) = \alpha$.



Why does continuity matter here?



Root-finding algorithms

solving $f(x) = 0$ → important to be able to solve

HARD

$$8 + \ln(x)\, x^2 e^{x \sin(x^3)} = 0$$

CAN'T SOLVE

FACT: if we can find an interval $[a, b]$ such that $(f(a) < 0$ and $f(b) > 0)$ OR $(f(a) > 0$ and $f(b) < 0)$