* in class attendance — open up

* online HW — accessible to all

check video in class mtg channel

- at cloud.fairmontstate.edu
  - usual login
  - (click "STUDENTS")

- at online HW ~ https://csmath.~

username: uca
pw : uca

EX: $\lim_{x\to 0} \dfrac{1}{1+\frac{1}{x}}$

$x=0$

$\dfrac{1}{1+\frac{1}{0}}$

uh oh

Generally

$\dfrac{1}{\infty} \sim 0$

$\dfrac{1}{1+\infty}$

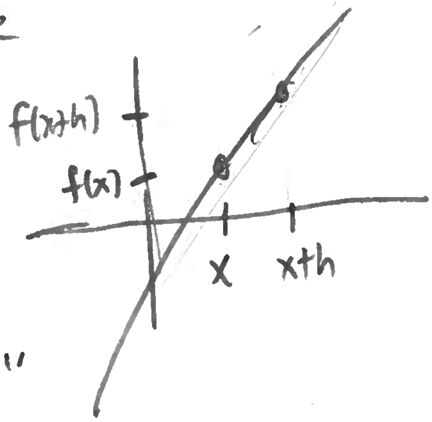
Note: $\dfrac{1}{1+\frac{1}{x}} = \left(\dfrac{1}{1+\frac{1}{x}}\right)\left(\dfrac{x}{x}\right)$

$= \dfrac{x}{x+1}$

So, $\lim_{x\to 0} \dfrac{1}{1+\frac{1}{x}} = \lim_{x\to 0} \dfrac{x}{x+1} = 0$

Ex: Given $f(x) = 3x - 1$, compute

$$\lim_{h\to 0} \underbrace{\frac{f(x+h) - f(x)}{h}}_{\text{"difference quotient"}}$$



$$= \lim_{h\to 0} \frac{[3(x+h) - 1] - [3x - 1]}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{3x} + 3h \cancel{- 1} \cancel{- 3x} \cancel{+ 1}}{h}$$

$$= \lim_{h\to 0} \frac{3\cancel{h}}{\cancel{h}} = \lim_{h\to 0} 3 = 3$$

Ex: $$\lim_{x\to 0} \frac{\sqrt{x+16} - 4}{x} = \lim_{x\to 0} \left(\frac{\sqrt{x+16} - 4}{x}\right)\underbrace{\left(\frac{\sqrt{x+16} + 4}{\sqrt{x+16} + 4}\right)}_{=1}$$

$x = 0$

$$\frac{\sqrt{16} - 4}{0} = \frac{4 - 4}{0} = \frac{0}{0}$$

more work to do

$$= \lim_{x\to 0} \frac{(\sqrt{x+16})^2 \cancel{+ 4\sqrt{x+16}} \cancel{- 4\sqrt{x+16}} - 16}{x(\sqrt{x+16} + 4)}$$

$$= \lim_{x\to 0} \frac{\cancel{x} \cancel{+16} \cancel{-16}}{\cancel{x}(\sqrt{x+16} + 4)}$$

$$= \lim_{x\to 0} \frac{1}{\sqrt{x+16} + 4}$$

$$= \frac{1}{\sqrt{16} + 4} = \frac{1}{8}$$

Ex: $\lim\limits_{x\to 5} \dfrac{\sqrt{x+2}-3}{x-5}$

$(a-b)(a+b) = a^2-b^2$

$x=5$

numerator not zero $\to \dfrac{\sqrt{7}-3}{0}$ DNE

denom $= 0$

$$\left(\frac{\sqrt{x+2}-3}{x-5}\right)\left(\frac{\sqrt{x+2}+3}{\sqrt{x+2}+3}\right)$$

$$= \frac{(x+2)-9}{(x-5)(\sqrt{x+2}+3)}$$

Recall: $\lim\limits_{x\to 0} \dfrac{\sin x}{x} = 1$

Similar: $\lim\limits_{x\to 0} \dfrac{\cos(x)-1}{x} = 0$

$$\lim_{\psi\to 0} \frac{1-\cos(3\psi)}{\sin(7\psi)} = \lim_{\psi\to 0} \underbrace{\left(\frac{7\psi}{\sin(7\psi)}\right)}_{\to 1}\left[\frac{1}{7\psi}\right]\underbrace{\left[\frac{1-\cos(3\psi)}{3\psi}\right]}_{\to 0}\left[3\psi\right]$$

$= \frac{3}{7}$

$\frac{3\psi}{7\psi} = \frac{3}{7}$

$$= 1 \cdot \left(\frac{3}{7}\right) \cdot 0$$

$$= 0$$

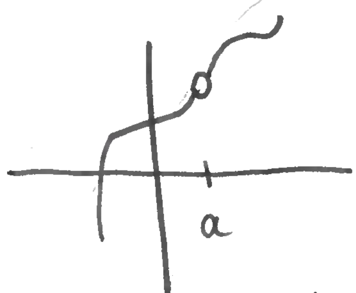
Continuity

We say a function is continuous at $x=a$ whenever:

① $f(a)$ exists,

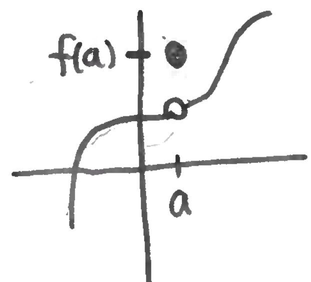
② $\lim_{x \to a} f(x)$ exists, and

③ $f(a) = \lim_{x \to a} f(x)$



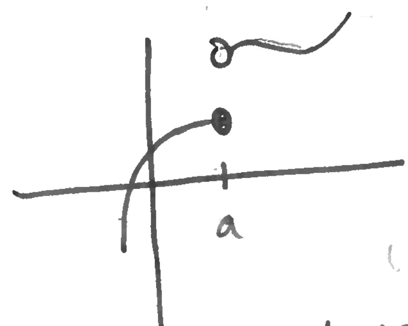
not ctn at $x=a$

(① is false)



not ctn at $x=a$

③ is false



not ctn at $x=a$

② fails

<u>informally:</u> "if you can draw graph w/o picking up pencil, then it is continuous"

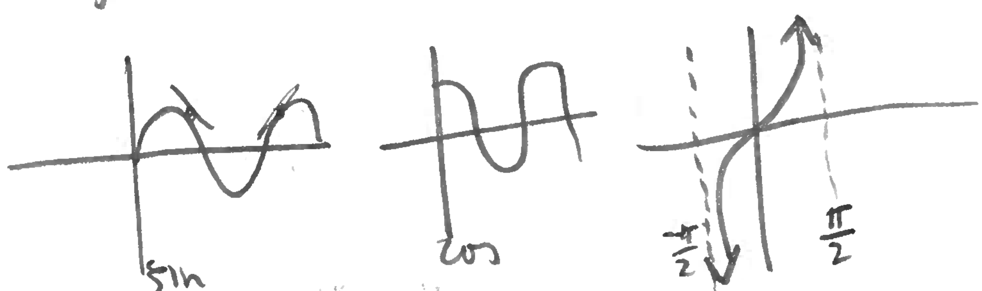
The following function types are ctn:

1. polynomials ~ ctn everywhere
2. rational functions – ctn on their domain

$\frac{p(x)}{q(x)}$ ← polynomials

$\frac{x^2+3}{x-9}$ $\frac{x^2-4}{x-2}$

3. ~~square~~ roots $\sqrt{x}, \sqrt[3]{x}, \sqrt[4]{x}, \ldots$
4. trig fncts on their domains



<u>Ex</u>: Consider $f(x) = \begin{cases} 2x+1; & x \le 5 \\ x^2+a; & x>5 \end{cases}$

unknown constant

Q: What value of a makes f ctn at $x=5$?

A: $\lim_{x\to5^-} f(x) = \lim_{x\to5^-} 2x+1 = 11$ ← $f(5)$

$\lim_{x\to5^+} f(x) = \lim_{x\to5^+} x^2+a = 5^2+a = 25+a$

Continuous: $25+a = 11 \to a = 11-25 = -14$