Ex: $\lim\limits_{x \to 4} \dfrac{\sqrt{x}-2}{x-4}$

$x=4$

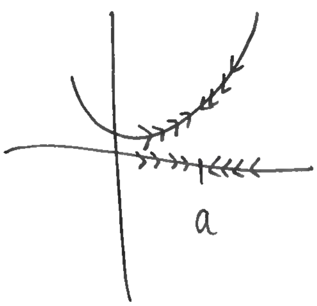
$\dfrac{\sqrt{4}-2}{4-4} = \dfrac{0}{0}$

more work to do

focus attention at 4 on the x-axis

$(\sqrt{2})(\sqrt{2}) = 2$

$\dfrac{\sqrt{2}}{\sqrt{2}} = 1$



Algebraic trick:

$$\frac{1}{\sqrt{2}} = \left(\frac{1}{\sqrt{2}}\right)(1) = \left(\frac{1}{\sqrt{2}}\right)\left(\frac{\sqrt{2}}{\sqrt{2}}\right) = \frac{\sqrt{2}}{2}$$

Recall: $(a-b)(a+b) = a^2 - b^2$

$a^2 - \cancel{ab} + \cancel{ab} - b^2$

$1 = \dfrac{\sqrt{x}+2}{\sqrt{x}+2}$

$$\frac{\sqrt{x}-2}{x-4} = \left(\frac{\sqrt{x}-2}{x-4}\right)(1)$$

$$= \left(\frac{\sqrt{x}-2}{x-4}\right)\left(\frac{\sqrt{x}+2}{\sqrt{x}+2}\right)$$

$$= \frac{(\sqrt{x})^2 - 2^2}{(x-4)(\sqrt{x}+2)} = \frac{\cancel{(x-4)}}{\cancel{(x-4)}(\sqrt{x}+2)} = \frac{1}{\sqrt{x}+2}$$

So,

$$\lim_{x \to 4} \frac{\sqrt{x}-2}{x-4} = \lim_{x \to 4} \frac{1}{\sqrt{x}+2} = \frac{1}{\sqrt{4}+2} = \frac{1}{4}$$

plug in $x=4$

**Theorem:** $\lim_{x \to a} P(x) = P(a)$ whenever $P(x)$ is a polynomial

**EX:** $\lim_{x \to 7} x^2 - 3x = 7^2 - 3(7)$

$= 49 - 21$

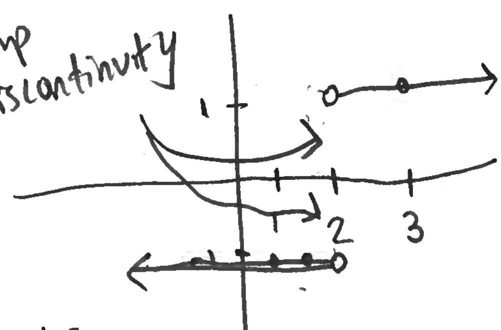
$= 28$

**EX:** $\lim_{x \to 2} \frac{|x^2-4|}{x-2}$ DOES NOT EXIST

$(x-2)(x+2) = x^2 - 4$

$$\frac{|x^2-4|}{x-2} = \frac{|(x-2)(x+2)|}{x-2} = \left(\frac{|x-2|}{x-2}\right)|x+2|$$

jump discontinuity




$\frac{|x-2|}{x-2}$

$x=3$

$\frac{|3-2|}{3-2} = \frac{1}{1} = 1$

$x=1$

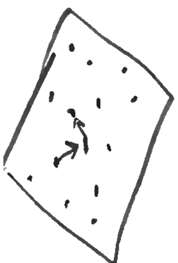
$\frac{|1-2|}{1-2} = \frac{1}{-1}$

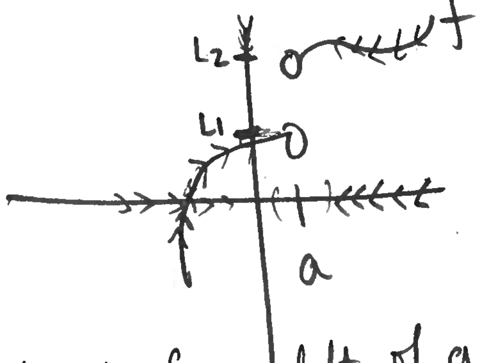
$x=1.5$

$\frac{|1.5-2|}{1.5-2} = \frac{|-0.5|}{-0.5} = -1$

# One-sided limits

Similar to limits, but restricts attention to only one side of $a$.

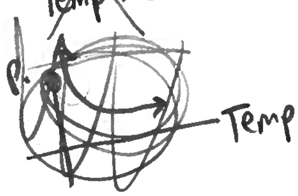





$\lim_{x \to a} f(x)$ DNE

What happens at absolute zero?

~~lim~~ Temp $\to 0$



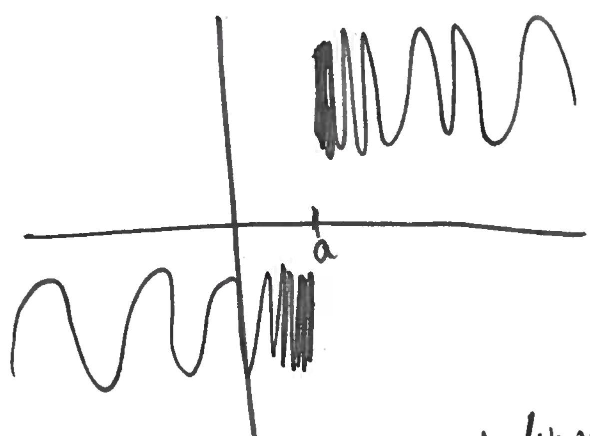

Limit from left of a

$$\lim_{x \to a^-} f(x) = L_1$$

Limit from right of a

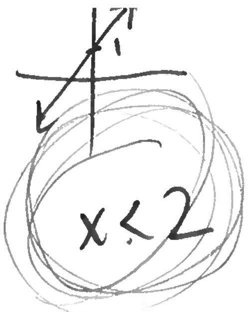
$$\lim_{x \to a^+} f(x) = L_2$$

EX:

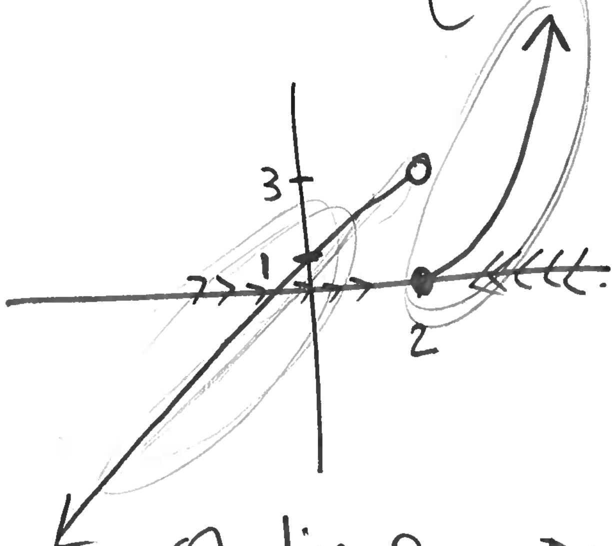



Has no left or right limits at $a$!

EX: Consider



$$f(x) = \begin{cases} x+1, & x<2 \\ x^2-4, & x \geq 2 \end{cases}$$



① $\lim\limits_{x \to 2} f(x)$ DNE

② $\lim\limits_{x \to 2^-} f(x) = \lim\limits_{x \to 2^-} x+1 = 2+1 = 3$

③ $\lim\limits_{x \to 2^+} f(x) = \lim\limits_{x \to 2^+} x^2-4 = 2^2-4 = 0$