~~Limits~~

Written HW 1 – "due" by 11:59PM tonight

— submit to Blackboard

("course content" ⇝ WHW1)

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online HW1 ⟶ WeBWork "due" by Wed at 11:59PM

$$\ln(a^b) = b\ln(a)$$

Ex:

$$8^{3x+7} = 3^{-5x+2}$$

⇓ plug both sides into $\ln(x)$

$$\ln(8^{3x+7}) = \ln(3^{-5x+2})$$

$$(3x+7)\underbrace{\ln(8)}_{\text{number}} = (-5x+2)\ln(3)$$

$$3x\ln(8) + 7\ln(8) = -5\ln(3)x + 2\ln(3)$$

$$3x\ln(8) + 5x\ln(3) = 2\ln(3) - 7\ln(8)$$

common factor of x

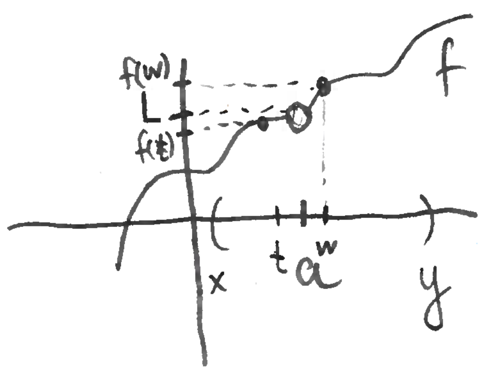
$$x\underbrace{\left[3\ln(8) + 5\ln(3)\right]}_{\text{number}} = \underbrace{2\ln(3) - 7\ln(8)}_{\text{number}}$$

$$x = \frac{2\ln(3) - 7\ln(8)}{3\ln(8) + 5\ln(3)}$$

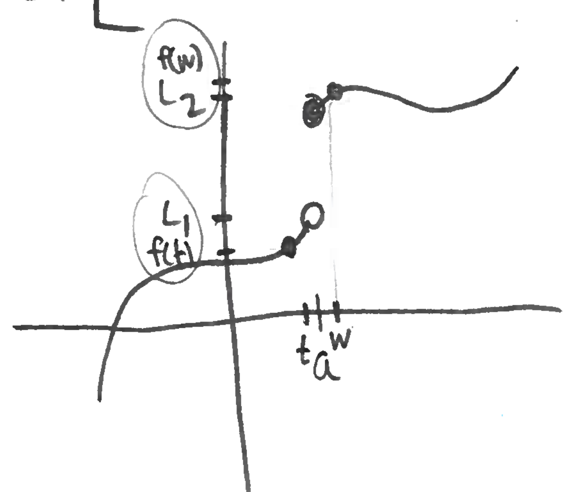
Def (limit): Let f be a fnct defined on an open interval (x,y) containing the point a, but maybe not defined at a itself.

If all values of f "get close to" height L as values of input to f "get close to" a, then we write

$$\lim_{t \to a} f(t) = L$$



$$\lim_{t \to a} f(t) = L$$



We see here:

"t is close to a"

$\downarrow$

"$f(t)$ close to L_1"

While

"w close to a"

$\downarrow$

"$f(w)$ close to L_2"

AND $L_2 \neq L_1$

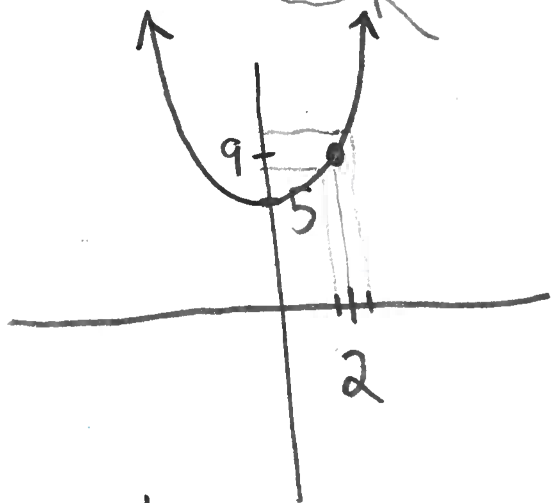
$\Rightarrow$ Therefore,

$\lim_{t \to a} f(t)$ DOES NOT EXIST (DNE)

EX: $f(x) = x^2 + 5$

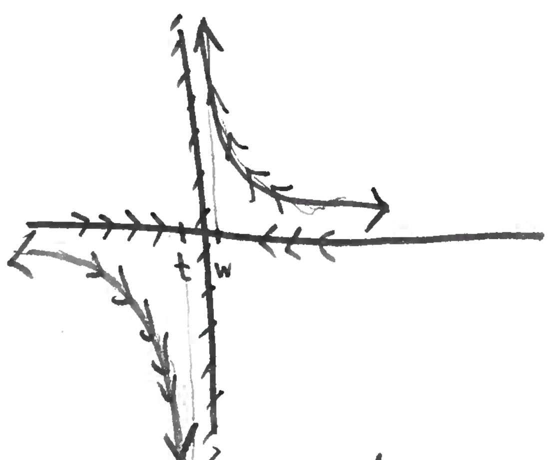
What is $\lim_{x \to 2} f(x)$?

$f(2) = 2^2 + 5$
$= 9$



$\lim_{x \to 2} f(x) = 9$

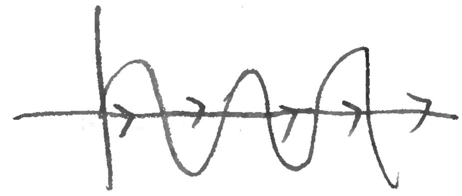
EX: $\lim_{x \to 0} \frac{1}{x}$



it looks like $\lim_{x \to 0} \frac{1}{x}$ could be $+\infty$ or $-\infty$

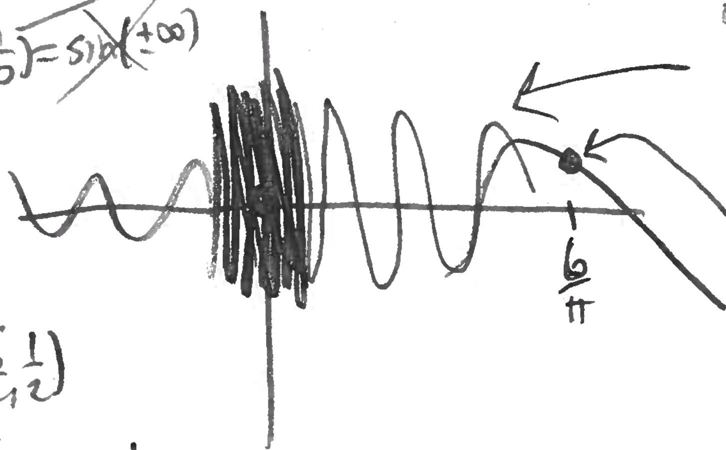
but we can't decide which

- in general if it is ∞ or $-\infty$
  we sometimes say limit <u>DNE</u>



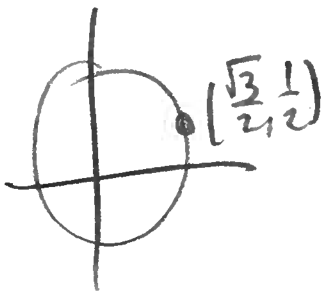
EX: $\lim_{x \to 0} \sin\left(\frac{1}{x}\right)$

$\sin\left(\frac{1}{0}\right) = \cancel{\sin}(\pm\infty)$



"topologist's sine curve"

$$\left(\frac{6}{\pi}, \sin\left(\frac{1}{\frac{6}{\pi}}\right)\right) = \left(\frac{6}{\pi}, \sin\left(\frac{\pi}{6}\right)\right) = \left(\frac{6}{\pi}, \frac{1}{2}\right)$$



$\lim_{x \to 0} \sin\left(\frac{1}{x}\right)$ DNE

$$\lim_{x \to \frac{6}{\pi}} \sin\left(\frac{1}{x}\right) = \frac{1}{2}$$

EX: $\lim_{x \to 0} \frac{\sin(x)}{x} = 1$

$\frac{\sin(0)}{0} = \frac{0}{0}$ ← more work to do

$3! = 3 \cdot 2 \cdot 1$

$4! = 4 \cdot 3 \cdot 2 \cdot 1$

Indian methematicians:

$$\sin(x) = \underbrace{x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \frac{x^9}{9!} - \frac{x^{11}}{11!} + \cdots}_{\text{Taylor series}}$$

Taylor series ← CALC 2

$$\frac{\sin(x)}{x} = \frac{x - \frac{x^3}{3!} + \frac{x^5}{5!} - \cdots}{x}$$

$$= \frac{x}{x} - \frac{x^3}{3!x} + \frac{x^5}{5!x} - \cdots$$

$$= 1 - \underbrace{\frac{x^2}{3!} + \frac{x^4}{5!} - \cdots}_{=0 \text{ when } x=0}$$

Ex: $\lim_{x\to 0} \frac{\sin(2x)}{x}$ ← mismatch

$$\boxed{\frac{2}{2} = 1}$$

$$\frac{\sin(2x)}{x} = \left(\frac{\sin(2x)}{x}\right)(1)$$

$$= \left(\frac{\sin(2x)}{2x}\right)2$$

So,

$$\lim_{x\to 0} \frac{\sin(2x)}{x} = \lim_{x\to 0} \overbrace{\frac{\sin(2x)}{2x}}^{1} \cdot 2$$

$$= 2$$