#6) Solve for θ in $[0, 2\pi)$:

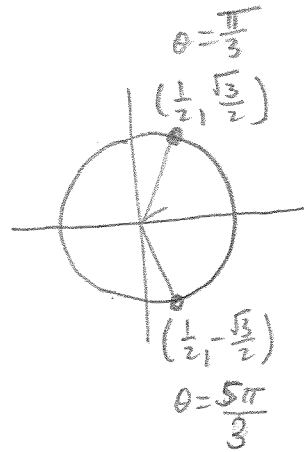$$2\cos(\theta) = 1$$

Soln: Div by 2,

$$\cos(\theta) = \frac{1}{2}$$

↓ from unit circle

$$\theta = \frac{\pi}{3}, \frac{5\pi}{3}$$

$\theta = \frac{\pi}{3}$

$\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$

$\left(\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$

$\theta = \frac{5\pi}{3}$

#17) Solve for θ in $[0, 2\pi)$: $2\sin(3\theta) = 1$

Soln: Divide by 2 to get
$$\sin(3\theta) = \frac{1}{2}$$

Let $\psi = 3\theta$, so it becomes
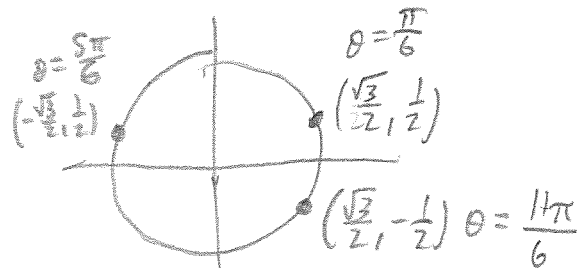
$$\sin(\psi) = \frac{1}{2}$$

↓ unit circle

$$\psi = \frac{\pi}{6}, \frac{5\pi}{6} \implies \begin{cases} 3\theta = \psi = \frac{\pi}{6} + 2n\pi \\ 3\theta = \psi = \frac{5\pi}{6} + 2n\pi \end{cases}$$

$\theta = \frac{\pi}{6}$

$\left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$

$\theta = \frac{5\pi}{6}$

$\left(-\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$

$\left(\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$ $\theta = \frac{11\pi}{6}$

$$\implies \begin{cases} \theta = \frac{\pi}{18} + \frac{2n\pi}{3} = \frac{\pi + 12n\pi}{18} \quad (i) \\ \theta = \frac{5\pi}{18} + \frac{2n\pi}{3} = \frac{5\pi + 12n\pi}{18} \quad (ii) \end{cases}$$

We need to find which n cause θ in $[0, 2\pi)$,

i.e, $0 \leq \theta < 2\pi$, or

$$0 \leq \theta < \frac{32\pi}{18}$$

For (i)

$n=3: \theta = \frac{\pi + 36\pi}{18} = \frac{37\pi}{18}$ ✗

$n=2: \theta = \frac{\pi + 24\pi}{18} = \frac{25\pi}{18}$ ✓

$n=1: \theta = \frac{\pi + 12\pi}{18} = \frac{13\pi}{18}$ ✓ $\Big\}$ good

$\theta = \frac{\pi + 12n\pi}{18} \rightsquigarrow$ $n=0: \theta = \frac{\pi}{18}$ ✓

$n=-1: \theta = \frac{\pi - 12\pi}{18} = \frac{-11\pi}{18}$ ✗

$\theta = \frac{5\pi + 12n\pi}{18} \longrightarrow$

$n=2: \frac{29\pi}{18}$ ✓ $n=3: \frac{41\pi}{18}$ ✗

$n=1: \frac{17\pi}{18}$ ✓

$n=0: \frac{5\pi}{18}$ ✓

$n=-1: -\frac{7\pi}{18}$ ✗

Therefore the solutions are

$$\theta = \frac{\pi}{18}, \frac{5\pi}{18}, \frac{13\pi}{18}, \frac{17\pi}{18}, \frac{25\pi}{18}, \frac{29\pi}{18}$$

#24) Solve for θ in $[0, 2\pi)$: $\tan(x) - 2\sin(x)\tan(x) = 0$

Soln: Factor the $\tan(x)$ to get

$$\tan(x)\left[1 - 2\sin(x)\right] = 0$$

$\theta = \frac{5\pi}{6}$

$\left(-\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$

$\left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right) \theta = \frac{\pi}{6}$

$(-1, 0)$ $\theta = \pi$

$(1, 0)$ $\theta = 0$

$\tan(x) = 0$

$\downarrow$

$x = 0, \pi$

$1 - 2\sin(x) = 0$

$\sin(x) = \frac{1}{2}$

$\downarrow$

$x = \frac{\pi}{6}, \frac{5\pi}{6}$