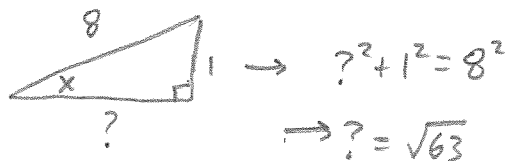


§9.3] #5] if $\sin(x) = \frac{1}{8}$ and x in QI , then find $\sin(2x)$, $\cos(2x)$, and $\tan(2x)$

(2)

Soln: From $\sin(x) = \frac{1}{8}$, draw a Δ :



Since x in QII ,
 $\cos(x) = -\frac{\sqrt{63}}{8}$

Therefore,

$$\begin{aligned}\sin(2x) &= 2\sin(x)\cos(x) = 2\left(\frac{1}{8}\right)\left(-\frac{\sqrt{63}}{8}\right) \\ &= \frac{-2\sqrt{63}}{64} = \frac{-\sqrt{63}}{32}\end{aligned}$$

$$\cos(2x) = \cos^2(x) - \sin^2(x)$$

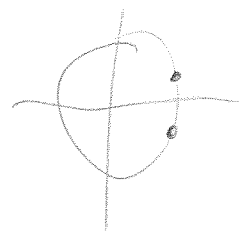
$$= \left(-\frac{\sqrt{63}}{8}\right)^2 - \left(\frac{1}{8}\right)^2 = \frac{63}{64} - \frac{1}{64} = \frac{62}{64} = \frac{31}{32}$$

$$\tan(2x) = \frac{\sin(2x)}{\cos(2x)} = \frac{-\frac{\sqrt{63}}{32}}{\frac{31}{32}} = -\frac{\sqrt{63}}{31}$$

#14] $\cos\left(-\frac{11\pi}{12}\right)$

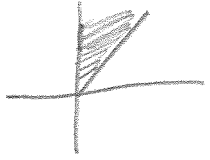
Soln: $-\frac{11\pi}{12} = \frac{\alpha}{2} \rightarrow \alpha = -\frac{11\pi}{6}$

Therefore, $\cos\left(-\frac{11\pi}{12}\right) = \cos\left(\frac{-\frac{11\pi}{6}}{2}\right) \stackrel{\text{identity}}{=} + \sqrt{\frac{1 + \cos\left(-\frac{11\pi}{6}\right)}{2}}$
in QI
 $= \sqrt{\frac{1 + \sqrt{3}/2}{2}}$



#22) if $\csc(x) = 7$ and x in QII , then find $\sin(\frac{x}{2})$, $\cos(\frac{x}{2})$, and $\tan(\frac{x}{2})$

(3)

Solu: if x in QII , then $\frac{\pi}{2} < x < \pi \Rightarrow \frac{\pi}{4} < \frac{x}{2} < \frac{\pi}{2}$
 in QI 
 \forall all in $QI \Rightarrow$ all \oplus

$$\frac{1}{\sin(x)} = \csc(x) = 7$$

$$\sin(x) = \frac{1}{7} \xrightarrow{\text{draw } \Delta} \begin{array}{c} 7 \\ \diagdown \\ x \\ \diagup \\ ? \end{array} \Rightarrow ?^2 + 1^2 = 7^2 \rightarrow \cos(x) = \frac{\sqrt{48}}{7}$$

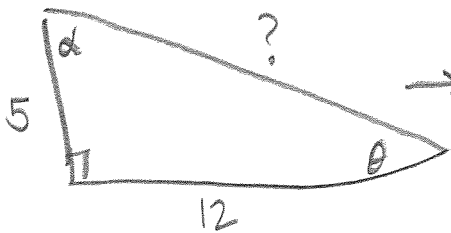
$$? = \sqrt{48}$$

$$\sin\left(\frac{x}{2}\right) = +\sqrt{\frac{1 - \cos(x)}{2}} = \sqrt{\frac{1 - \frac{\sqrt{48}}{7}}{2}}$$

$$\cos\left(\frac{x}{2}\right) = +\sqrt{\frac{1 + \cos(x)}{2}} = \sqrt{\frac{1 + \frac{\sqrt{48}}{7}}{2}}$$

$$\tan\left(\frac{x}{2}\right) = \frac{\sin\left(\frac{x}{2}\right)}{\cos\left(\frac{x}{2}\right)} = \frac{\sqrt{\frac{1 - \frac{\sqrt{48}}{7}}{2}}}{\sqrt{\frac{1 + \frac{\sqrt{48}}{7}}{2}}}$$

#25)



$$\begin{aligned} 12^2 + 5^2 &= ?^2 \\ 144 + 25 &= ?^2 \\ ? &= \sqrt{169} = 13 \end{aligned}$$

$$\sin(2\alpha) = 2\sin(\alpha)\cos(\alpha) = 2\left(\frac{12}{13}\right)\left(\frac{5}{13}\right) = \frac{120}{13^2}$$

$$\cos(2\alpha) = \cos^2(\alpha) - \sin^2(\alpha) = \left(\frac{5}{13}\right)^2 - \left(\frac{12}{13}\right)^2$$

$$\tan(2\alpha) = \frac{\sin(2\alpha)}{\cos(2\alpha)} = \frac{\frac{120}{13^2}}{\frac{5^2 - 12^2}{13^2}} = \frac{120}{119}$$

$$25 - 144$$

$$\begin{array}{r} 314 \\ 144 \\ \hline 25 \\ \hline 119 \end{array}$$

#36) Prove identity

$$\cot(x) - \tan(x) = 2\cot(2x)$$

Soln: Start with right:

$$2\cot(2x) = \frac{2\cos(2x)}{\sin(2x)}$$

$$= \cancel{2} \left[\frac{\cos^2(x) - \sin^2(x)}{\cancel{2}\cos(x)\sin(x)} \right]$$

$$= \frac{\cancel{\cos^2}(x)}{\cancel{\cos}(x)\cancel{\sin}(x)} - \frac{\cancel{\sin^2}(x)}{\cancel{\cos}(x)\cancel{\sin}(x)}$$

$$= \frac{\cos(x)}{\sin(x)} - \frac{\sin(x)}{\cos(x)}$$

$$= \cot(x) - \tan(x), \text{ as was to be shown. } \blacksquare$$

#58) Prove identity

$$(\sin^2(x) - 1)^2 = \cos(2x) + \sin^4(x)$$

Soln: From left,

$$(\sin^2(x) - 1)^2 = \sin^4(x) - \underbrace{2\sin^2(x) + 1}_{= \cos(2x) \text{ by identity}}$$

$$= \sin^4(x) + \cos(2x),$$

as was to be shown. \blacksquare