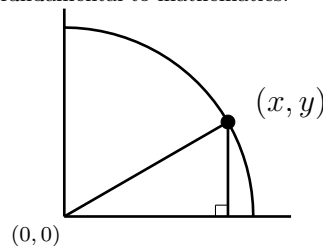


Points: _____ out of 20 Arizona Rubric Score: _____/4 Name: *Write your name on back!*

Outcome 4: Develop and evaluate mathematical arguments and proofs to recognize that reasoning and proof are fundamental to mathematics.

4 : 18 – 20, 3 : 14 – 17, 2 : 10 – 13, 1 : 6 – 9, 0 : 0 – 5

- (1) (4pt) Explain why the identity $\sin^2(\theta) + \cos^2(\theta) = 1$ is true. The diagram on the right may be useful to consider.



- (2) (4pt) Derive the identity $1 + \cot^2(\theta) = \csc^2(\theta)$ from the identity $\sin^2(\theta) + \cos^2(\theta) = 1$.

- (3) (4pt) Prove the following identity.

$$\sin(\theta) \csc(\theta) - \cos^2(\theta) = \sin^2(\theta)$$

- (4) (4pt) There are mistakes in the following “proof” of the identity

$$\frac{1}{1 - \cos(\theta)} + \frac{1}{1 + \cos(\theta)} = \frac{2}{\sin^2(\theta)}. \text{ Find two of them and explain why they are wrong.}$$

Proof:

$$\begin{aligned} \frac{1}{1 - \cos(\theta)} + \frac{1}{1 + \cos(\theta)} &= \frac{1}{\sin(\theta)} + \frac{1}{\sin(\theta)} \\ &= \frac{2}{\sin^2(\theta)} \end{aligned}$$

- (5) (4pt) The sum identity for cosine, $\cos(\alpha + \beta) = \cos(\alpha)\cos(\beta) - \sin(\alpha)\sin(\beta)$, has been derived from the fundamental identities and the unit circle. Use this identity to simplify the following expression. Your final answer should be a function of x and not include any references to $\frac{\pi}{2}$.

$$\cos\left(x + \frac{\pi}{2}\right) =$$