Written HW13 – MATH 1540 Fall 2020
Due by 14 November for timely completion credit

The so-called Chebyshev polynomials (of the first kind), written $T_n(x)$, are given by the formula

$$T_n(x) = \cos(n \arccos(x)), \tag{1}$$

for $n = 0, 1, 2, \ldots$ and for $-1 \leq x \leq 1$. In this homework, you will derive the formulas for some of the Chebyshev polynomials.

For the following problems your knowledge of inverse trigonometric functions as well as the double angle identity

$$\cos(2x) = \cos^2(x) - \sin^2(x)$$

and the sum of angles identity

$$\cos(a + b) = \cos(a)\cos(b) - \sin(a)\sin(b)$$

will be needed.

1. Write down equation (1) for $n = 1$. Simplify until you arrive at a traditional polynomial in the variable x.
2. Write down equation (1) for $n = 2$. Simplify until you arrive at a traditional polynomial in the variable x.
3. Write down equation (1) for $n = 3$. Simplify until you arrive at a traditional polynomial in the variable x.

(*note: Chebyshev polynomials are traditonally used in "approximation theory", where they are utilized in finding polynomial approximations of more complicated functions — this ends up being important for many computer simulations of scientific phenomena*)