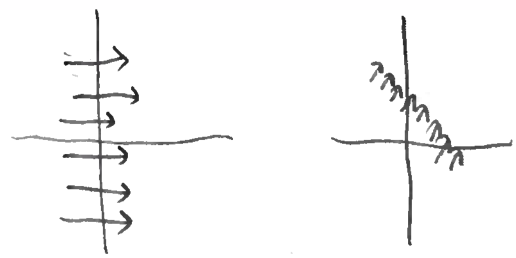
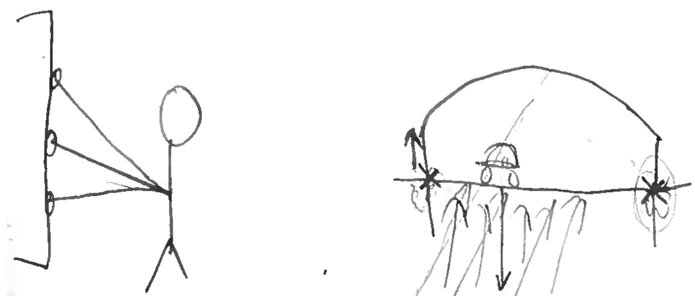
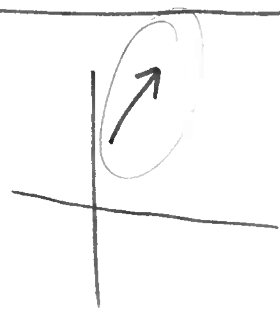
Vectors

A vector is an arrow ~ it has a length ("magnitude" or "norm") and a direction.





Vectors in plane (2 dimensional)



Physics type

arrows

good for physical intuition

CompSci Type

lists of numbers

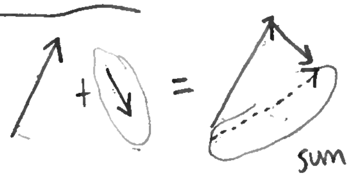
good for computation

$\langle 1, 2 \rangle$

1 unit in x-dir

2 units in y-dir

Adding vectors

Physics: 

sum

CS way: $\langle a,b\rangle + \langle c,d\rangle = \langle a+c, b+d\rangle$

Scaler multiplication

numbers

~~Let α be a number.~~

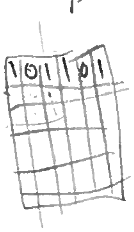
Physics: $2(\nearrow) = \nearrow$ $\quad \frac{1}{2}(\nearrow) = \nearrow$ $\quad -(\nearrow) = \swarrow$

CS: $2\langle a,b\rangle = \langle 2a, 2b\rangle$ $\quad \frac{1}{2}\langle a,b\rangle = \langle \frac{a}{2}, \frac{b}{2}\rangle$ $\quad -\langle a,b\rangle = \langle -a, -b\rangle$

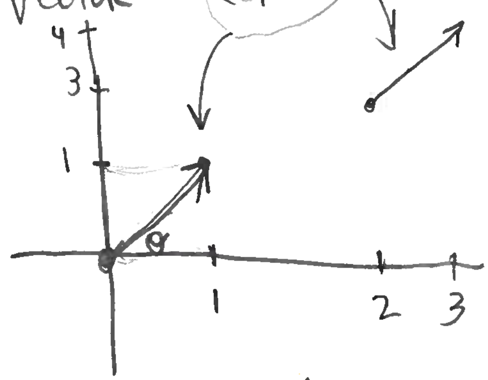
Dot Product

$\langle a,b\rangle \cdot \langle c,d\rangle = ac + bd$ (turns out to be related to angle b/w vectors)

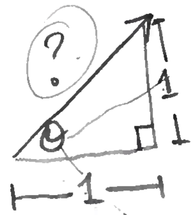
Componentwise mult. of vecters is called Hadamard product
(used in JPEG compression)



Ex: Vector $\langle 1, 1 \rangle$

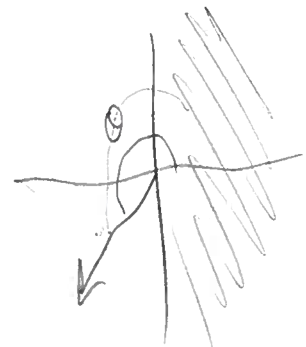


length of vector:



$\rightarrow \quad 1^2 + 1^2 = ?^2$

$? = \sqrt{2}$



$\underbrace{\text{length of}}$ $\langle 1,1 \rangle$ is $\sqrt{2}$

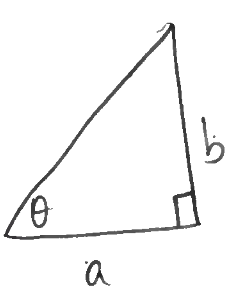
$\|\langle 1,1 \rangle\| = \sqrt{2}$

Direction of $\langle 1,1 \rangle$ is the angle θ in picture.

$\tan(\theta) = \frac{1}{1}$

$\theta = \arctan(1) = \frac{\pi}{4} = 45^\circ$

General:



$$\boxed{\begin{array}{l} \text{Length of } \langle a,b \rangle = \|\langle a,b \rangle\| = \sqrt{a^2+b^2} \\ \text{direction of } \langle a,b \rangle = \begin{cases} \arctan\left(\frac{b}{a}\right); & (a,b) \text{ is in QI or QIV} \\ \pi + \arctan\left(\frac{b}{a}\right); & (a,b) \text{ is in QII or QIII} \end{cases} \end{array}}$$

Ex: For $\vec{a} = \langle 3, -2 \rangle$ and $\vec{b} = \langle -1, -1 \rangle$

compute...

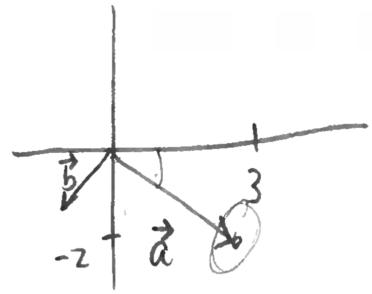
$2\vec{a} - \vec{b}$, $3\vec{a} + 5\vec{b}$, $\vec{a} \cdot \vec{b}$

$\|\vec{a}\|$

direction of $\vec{a}$

$\|\vec{a} + \vec{b}\|$

direction of $\vec{a} + \vec{b}$



Soln: $2\vec{a} - \vec{b} = 2\langle 3, -2 \rangle - \langle -1, -1 \rangle$

$= \langle 6, -4 \rangle + \langle 1, 1 \rangle$

$= \langle 7, -3 \rangle$

$3\vec{a} + 5\vec{b} = 3\langle 3, -2 \rangle + 5\langle -1, -1 \rangle$

$= \langle 9, -6 \rangle + \langle -5, -5 \rangle$

$= \langle 4, -11 \rangle$

$\vec{a} \cdot \vec{b} = \langle 3, -2 \rangle \cdot \langle -1, -1 \rangle = 3(-1) + (-2)(-1)$

$= -3 + 2 = -1$

$\|\vec{a}\| = \|\langle 3, -2 \rangle\| = \sqrt{3^2 + (-2)^2} = \sqrt{9+4} = \sqrt{13}$

direction of $\vec{a} = \arctan\left(\frac{-2}{3}\right) \approx -0.59\,\text{rad} \approx -33.69^\circ$

$\|\vec{a} + \vec{b}\| = \|\langle 3, -2 \rangle + \langle -1, -1 \rangle\| = \|\langle 2, -3 \rangle\| = \sqrt{2^2 + (-3)^2} = \sqrt{13}$

dir of $\vec{a} + \vec{b}$ = dir of $\langle 2, -3 \rangle = \arctan\left(\frac{-3}{2}\right) = -0.98\,\text{rad} \approx -56.3^\circ$