Ex: Solve

Double angle identity

$\sin(2x) = 2\sin(x)\cos(x)$

$$\sin(2x) - \cos(x) = 0$$

Soln: Apply dbl angle identity:

$$2\sin(x)\cos(x) - \cos(x) = 0$$

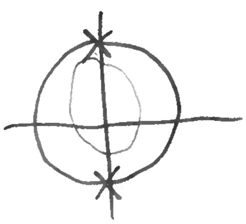
$$\cos(x)\left[2\sin(x) - 1\right] = 0$$

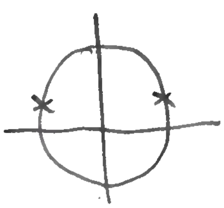
$ab = 0$

$a = 0$ OR $b = 0$

OR

$\cos(x) = 0$ $2\sin(x) - 1 = 0$





From unit circle, we see that $\cos(x) = 0$ when $x = \underbrace{\frac{\pi}{2}, \frac{3\pi}{2}}_{\text{"fundamental"}}$

$\Rightarrow$ general soln

$$x = \frac{\pi}{2} + 2k\pi, \ k \in \mathbb{Z}$$

$$x = \frac{3\pi}{2} + 2k\pi, \ k \in \mathbb{Z}$$

$$2\sin(x) = 1$$

$$\sin(x) = \frac{1}{2}$$

from 13 November notes

$$x = \frac{\pi}{6} + 2\pi k, \ k \in \mathbb{Z}$$

$$x = \frac{5\pi}{6} + 2\pi k, \ k \in \mathbb{Z}$$

Ex: Solve $\dfrac{\sin(2x)}{\sec^2(x)} = 0$

Soln: Apply dbl angle identity to top
+ def of secant to bottom

$$\frac{2\sin(x)\cos(x)}{\frac{1}{\cos^2(x)}} = 0$$

$\downarrow$ flip + mult. $\quad \dfrac{\frac{a}{b}}{\frac{c}{d}} = \dfrac{a}{b} \cdot \dfrac{d}{c}$

$$2\sin(x)\cos^3(x) = 0$$

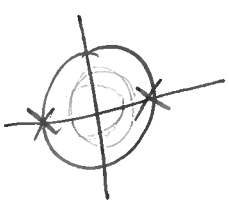
$\downarrow$ div by 2

$$\sin(x)\cos^3(x) = 0$$

OR

$\sin(x) = 0$

$\downarrow$



From unit circle we get fundamental solns of $x = 0$, $x = \pi$

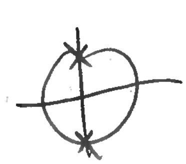
$\downarrow$

So general soln is
$x = 0 + 2\pi k$, $k \in \mathbb{Z}$
$x = \pi + 2\pi k$, $k \in \mathbb{Z}$

$\cos^3(x) = 0$

$\downarrow$ take $\sqrt[3]{\ }$ of both sides

$\cos(x) = 0$



$\downarrow$ prev. example

$x = \frac{\pi}{2} + 2\pi k$, $k \in \mathbb{Z}$

$x = \frac{3\pi}{2} + 2\pi k$, $k \in \mathbb{Z}$

Ex: Solve

$$\sin^2(x)\underline{\left(1-\sin^2(x)\right)}+\cos^2(x)\underline{\left(1-\sin^2(x)\right)}=0$$

Pythagorean identity

$\sin^2(x)+\cos^2(x)=1$

Soln: Factor

$$\left(1-\sin^2(x)\right)\underbrace{\cancel{\left(\sin^2(x)+\cos^2(x)\right)}}_{=1}=0$$

$$1-\sin^2(x)=0$$

$$\sin^2(x)=1$$

$$\downarrow \sqrt{\ }$$

$$\sin(x)=\pm\sqrt{1}=\pm 1$$

$\sin(x)=1$

Fundamental: $x=\frac{\pi}{2}$

General: $x=\frac{\pi}{2}+2\pi k,\ k\in\mathbb{Z}$

$\sin(x)=-1$

Fundamental: $x=\frac{3\pi}{2}$

General: $x=\frac{3\pi}{2}+2k\pi,\ k\in\mathbb{Z}$

Ex: Solve

$$8\sin^2(x) + 6\sin(x) + 1 = 0$$

Soln: This equation is "quadratic in form", meaning if we define

$$W = \sin(x)$$

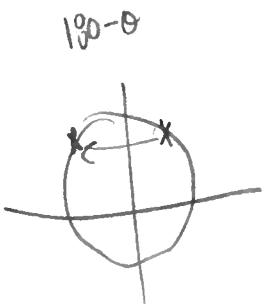
then it becomes a quadratic equation in W,

$$8W^2 + 6W + 1 = 0$$

$\downarrow$ QF $\left(\begin{array}{c} ax^2+bx+c=0 \\ x = \dfrac{-b \pm \sqrt{b^2-4ac}}{2a} \end{array} \right)$

$$W = \frac{-6 \pm \sqrt{6^2 - 4(8)(1)}}{16} = -\frac{6}{16} \pm \frac{\sqrt{4}}{16}$$

$\frac{2}{16}$

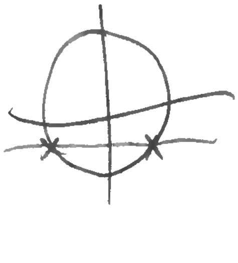


$$= -\frac{3}{8} \pm \frac{1}{8}$$

$\ominus$ / $\oplus$

$W = -\frac{4}{8} = -\frac{1}{2}$ $\qquad$ $W = -\frac{2}{8} = -\frac{1}{4}$

$\downarrow W = \sin x$ $\qquad$ $\sin(x) = -\frac{1}{4}$ ← NOT ON unit circle



$$\sin(x) = -\frac{1}{2}$$

Fundamental: $\frac{7\pi}{6}, \frac{11\pi}{6}$

General: $x = \frac{7\pi}{6} + 2\pi k$

$x = \frac{11\pi}{6} + 2\pi k$

only option

$\boxed{\begin{array}{l} x = \arcsin(-1/4) \approx -0.253 \quad \text{in QIV} \\ \text{other one in QIII: } x = -\pi + 0.253 \\ \qquad \approx -2.889 \end{array}}$

Fundamental

General: $x = -0.253 + 2\pi k$, $x = -2.889 + 2\pi k$

Ex: Solve

$$\sin(\theta) = \frac{\sqrt{3}}{2} \text{ for } \theta \text{ in } [0, 2\pi).$$

Soln: Method: first find general soln,
then pick solns in correct range

Fundamental: from unit circle

$$\theta = \frac{\pi}{3}, \frac{2\pi}{3}$$

General:

$$\theta = \frac{\pi}{3} + 2\pi k, \quad k \in \mathbb{Z} = \{\ldots, -1, 0, 1, \ldots\}$$

$$\theta = \frac{2\pi}{3} + 2\pi k, \quad k \in \mathbb{Z}$$

Which values of k give us a θ in the interval $[0, 2\pi]$?

$2\pi = \frac{6\pi}{3}$

$$\theta = \frac{\pi}{3} + 2\pi k = \frac{\pi + 6\pi k}{3} \longrightarrow$$

common denom.

$k = 2$: ↑ x

$k = 1$: $\frac{7\pi}{3}$ x too big

$k = 0$: $\frac{\pi}{3}$ ✓

$k = -1$: $-\frac{5\pi}{3}$ x too negative

$k = -2$: ↓ x

$$\theta = \frac{2\pi}{3} + 2\pi k = \frac{2\pi + 6\pi k}{3} \longrightarrow$$

$k = 1$: $8\pi/3$ x too big

$k = 0$: $2\pi/3$ ✓

$k = -1$: $-4\pi/3$ x too negative

We get two valid solns in $[0, 2\pi]$:

$$\theta = \frac{\pi}{3}, \frac{2\pi}{3}$$