

# Trigonometric Equations

Goal: find the solution(s)

Ex: Solve  $\sin(x) = \frac{1}{2}$

Soln: From unit circle, we get two "fundamental solutions" which are  $\frac{\pi}{6}$  and  $\frac{5\pi}{6}$ .  $(-\frac{\sqrt{3}}{2}, \frac{1}{2})$

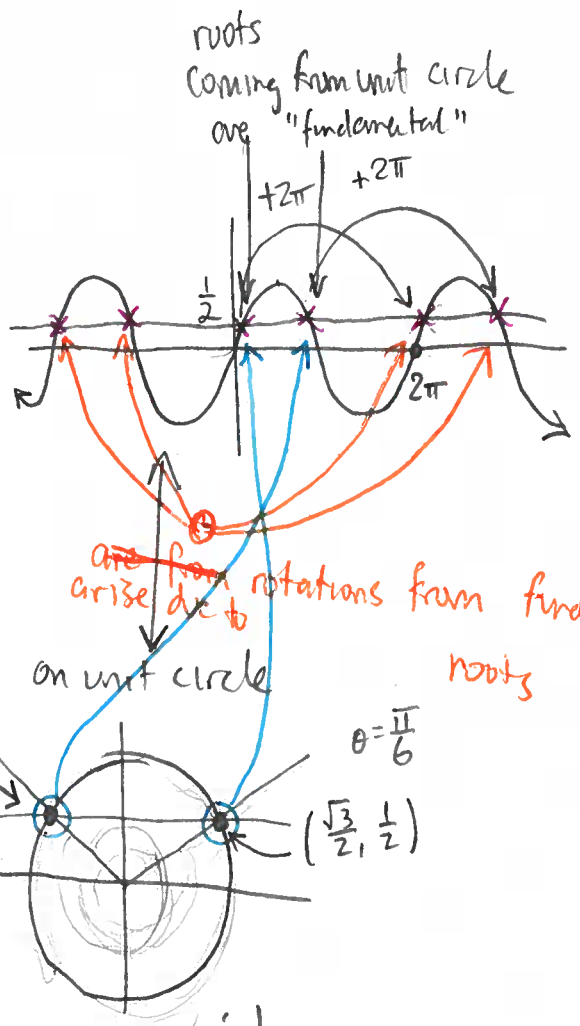
The rest of solns come from full rotations from those two.  
How to write?

Let  $k$  be "an arbitrary integer", then all solns can be written in form

$k$   
 $k$   
 $k$   
 $k$

$$\begin{cases} x = \frac{\pi}{6} + 2\pi k, & k \text{ is an integer} \\ x = \frac{5\pi}{6} + 2\pi k, & k \in \mathbb{Z} \end{cases}$$

↑  
"element of"



$$\mathbb{Z} = \{\dots, -3, -2, -1, 0, 1, 2, 3, \dots\}$$

"z" used b/c German word for "number" is Zahlen

$$k = \dots, -1, 0, 1, \dots$$

Ex: Solve  $\cos(\theta) = -\frac{\sqrt{3}}{2}$ .

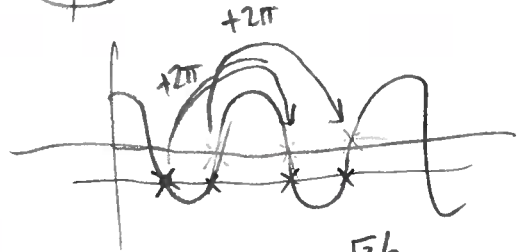
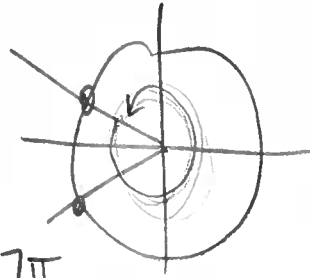
Soln: From unit circle:

$$\theta = \frac{5\pi}{6} \text{ and } \theta = \frac{7\pi}{6}$$

Then the general soln is

$$\theta = \frac{5\pi}{6} + 2\pi k, k \in \mathbb{Z}$$

$$\theta = \frac{7\pi}{6} + 2\pi k, k \in \mathbb{Z}$$



Ex: Soln  $\tan(\theta) = 1$ .



From unit circle,

$$\theta = \frac{\pi}{4}, \theta = \frac{5\pi}{4}$$



general soln

$$\begin{cases} \theta = \frac{\pi}{4} + 2\pi k, k \in \mathbb{Z} \\ \theta = \frac{5\pi}{4} + 2\pi k, k \in \mathbb{Z} \end{cases}$$

