Ex: Given that $\tan(\theta) = -\frac{1}{5}$ and $\sin(\theta) < 0$,

Compute

$\sin(\theta + \frac{\pi}{4})$

$\cos(\theta - \frac{\pi}{3})$

$\tan(\theta + \frac{\pi}{6})$

$\sec(2\theta) = \sec(\theta + \theta)$

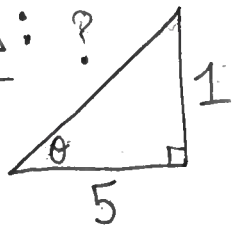
$\sin > 0$	$\sin > 0$
$\cos < 0$	$\cos > 0$
$\tan < 0$	$\tan > 0$
$\sin \not< 0$	$\sin \not> 0$
$\cos < 0$	$\cos > 0$
$\tan > 0$	$\tan < 0$

Soln: Since $\tan(\theta) < 0$, θ is in QII or QIV.

Since $\sin(\theta) < 0$, θ is in QIII or QIV.

Therefore, θ is in QIV.

Draw a Δ:



$\rightarrow 5^2 + 1^2 = ?^2$

$? = \sqrt{26}$

Compute

$$\sin(\underset{a}{\theta} + \underset{b}{\frac{\pi}{4}}) = \sin(\theta)\cos(\tfrac{\pi}{4}) + \cos(\theta)\sin(\tfrac{\pi}{4})$$

$$= \left(-\frac{1}{\sqrt{26}}\right)\left(\frac{\sqrt{2}}{2}\right) + \left(\frac{5}{\sqrt{26}}\right)\left(\frac{\sqrt{2}}{2}\right)$$

$$= \frac{-\sqrt{2} + 5\sqrt{2}}{2\sqrt{26}} = \frac{4\sqrt{2}}{2\sqrt{26}} = \frac{2\sqrt{2}}{\sqrt{26}}$$

Compute

$$\cos(\underset{a}{\theta} - \underset{b}{\frac{\pi}{3}}) = \cos(\theta)\cos(\tfrac{\pi}{3}) + \sin(\theta)\sin(\tfrac{\pi}{3})$$

$$= \left(\frac{5}{\sqrt{26}}\right)\left(\frac{1}{2}\right) + \left(-\frac{1}{\sqrt{26}}\right)\left(\frac{\sqrt{3}}{2}\right) = \frac{5 - \sqrt{3}}{2\sqrt{26}}$$

Compute

$$\tan\left(\theta+\frac{\pi}{6}\right)=\frac{\sin\left(\theta+\frac{\pi}{6}\right)}{\cos\left(\theta+\frac{\pi}{6}\right)}=\frac{\sin(\theta)\cos\left(\frac{\pi}{6}\right)+\cos(\theta)\sin\left(\frac{\pi}{6}\right)}{\cos(\theta)\cos\left(\frac{\pi}{6}\right)-\sin(\theta)\sin\left(\pi/6\right)}$$

$$=\frac{\left(-\frac{1}{\sqrt{26}}\right)\left(\frac{\sqrt{3}}{2}\right)+\left(\frac{5}{\sqrt{26}}\right)\left(\frac{1}{2}\right)}{\left(\frac{5}{\sqrt{26}}\right)\left(\frac{\sqrt{3}}{2}\right)-\left(-\frac{1}{\sqrt{26}}\right)\left(\frac{1}{2}\right)}$$

$$\frac{\frac{a}{b}}{\frac{c}{d}}=\frac{a}{b}\cdot\frac{d}{c}$$

$$=\frac{\left(\frac{-\sqrt{3}+5}{2\sqrt{26}}\right)}{\left(\frac{5\sqrt{3}+1}{2\sqrt{26}}\right)}$$

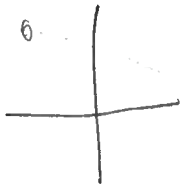
$$=\left(\frac{-\sqrt{3}+5}{2\sqrt{26}}\right)\left(\frac{2\sqrt{26}}{5\sqrt{3}+1}\right)=$$

$$=\frac{-\sqrt{3}+5}{5\sqrt{3}+1}$$

Ex: Given $\cos(\alpha) = -\frac{1}{9}$ and $\sin(\beta) = \frac{1}{4}$

β is in QII

$\left(\pi < \alpha < \frac{3\pi}{2}\right)$ α is in QIII

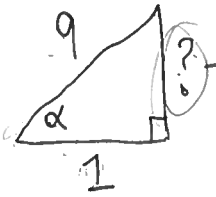


Compute

$\sin(\alpha - \beta)$

$\cos(\alpha + \beta)$

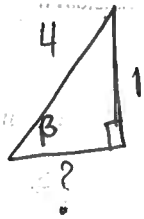
Soln: Draw a Δ for $\cos(\alpha) = -\frac{1}{9}$



$1^2 + ?^2 = 9^2$

$? = \sqrt{80}$

Draw Δ for $\sin(\beta) = \frac{1}{4}$



$?^2 + 1^2 = 4^2$

$? = \sqrt{15}$

Compute

$$\sin(\alpha - \beta) = \sin(\alpha)\cos(\beta) - \cos(\alpha)\sin(\beta)$$

$$= \left(-\frac{\sqrt{80}}{9}\right)\left(-\frac{\sqrt{15}}{4}\right) - \left(-\frac{1}{9}\right)\left(\frac{1}{4}\right)$$

$$= \frac{\sqrt{80}\sqrt{15} + 1}{36}$$

Compute

$$\cos(\alpha + \beta) = \cos(\alpha)\cos(\beta) - \sin(\alpha)\sin(\beta)$$

$$= \left(-\frac{1}{9}\right)\left(-\frac{\sqrt{15}}{4}\right) - \left(-\frac{\sqrt{80}}{9}\right)\left(\frac{1}{4}\right)$$

$$= \frac{\sqrt{15} + \sqrt{80}}{36}$$

Ex: Given $\csc(\theta) = -6$ and $\tan(\psi) = 12$

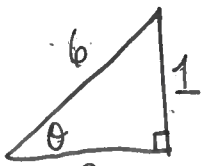
θ is in QIV $\rightarrow \frac{3\pi}{2} < \theta < 2\pi$

ψ is in QIII

Compute $\sin(\theta+\psi)$ and $\cos(\theta+\psi)$ and $\tan(\theta+\psi)$

Soln:

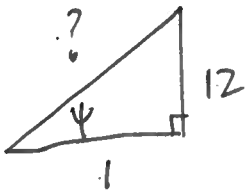
Draw Δ for $\csc(\theta) = -6 = \frac{-6}{1}$



$\rightarrow ?^2 + 1^2 = 6^2$

$? = \sqrt{35}$

Draw Δ for $\tan(\psi) = 12 = \frac{12}{1}$



$1^2 + 12^2 = ?^2$

$\sqrt{145} = ?$

Compute

$$\sin(\theta+\psi) = \sin(\theta)\cos(\psi) + \cos(\theta)\sin(\psi)$$

$$= \left(-\frac{1}{6}\right)\left(-\frac{1}{\sqrt{145}}\right) + \left(\frac{\sqrt{35}}{6}\right)\left(-\frac{12}{\sqrt{145}}\right)$$

$$= \frac{1 - 12\sqrt{35}}{6\sqrt{145}}$$

Compute

$$\cos(\theta+\psi) = \cos(\theta)\cos(\psi) - \sin(\theta)\sin(\psi)$$

$$= \left(\frac{\sqrt{35}}{6}\right)\left(-\frac{1}{\sqrt{145}}\right) - \left(-\frac{1}{6}\right)\left(-\frac{12}{\sqrt{145}}\right)$$

$$= \frac{-\sqrt{35} - 12}{6\sqrt{145}}$$

Compute

$$\tan(\theta+\psi) = \frac{\sin(\theta+\psi)}{\cos(\theta+\psi)} = \frac{\frac{1-12\sqrt{35}}{6\sqrt{145}}}{\frac{-\sqrt{35}-12}{6\sqrt{145}}} = \frac{1-12\sqrt{35}}{-\sqrt{35}-12}$$