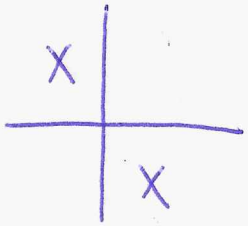
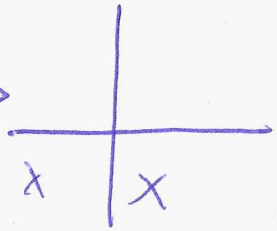
Ex: if $\tan(x) = -\frac{1}{8}$, and if $\sin(x) < 0$,

find $\sin(2x)$, $\cos(2x)$, and $\tan(2x)$

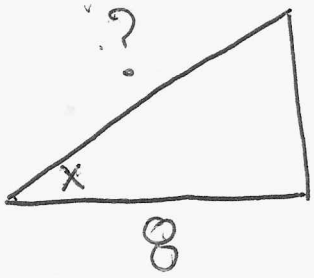
Soln:

since $\tan(x) < 0$ → 

x is in QII or QIV ⇒ x in QIV

since $\sin(x) < 0$ → 

x is in QIII or ~~QIV~~



$\rightarrow 8^2 + 1^2 = ?^2$

$? = \sqrt{65}$

$\sin(2x) = 2\sin(x)\cos(x)$ ← dbl angle identity

$= 2\left(-\frac{1}{\sqrt{65}}\right)\left(\frac{8}{\sqrt{65}}\right)$

$= \frac{-16}{65}$

$$\cos(2x) = \cos^2(x) - \sin^2(x)$$

$$= \left(\frac{8}{\sqrt{65}}\right)^2 - \left(-\frac{1}{\sqrt{65}}\right)^2$$

$$= \frac{64}{65} - \frac{1}{65}$$

$$= \frac{63}{65}$$

$$\tan(2x) = \frac{\sin(2x)}{\cos(2x)} = \frac{-\frac{16}{65}}{63/65} = \left(-\frac{16}{65}\right)\left(\frac{65}{63}\right)$$

$$= -\frac{16}{63}$$

$\frac{\cos}{\sin}$ $\frac{\sin}{\cos}$

Ex: Prove identity $\cot(x) - \tan(x) = 2\cot(2x)$ $\qquad \frac{a+b}{c} = \frac{a}{c} + \frac{b}{c}$

Soln: Start w/ right side:

$$2\cot(2x) = \frac{2\cos(2x)}{\sin(2x)} = \frac{2(\cos^2(x) - \sin^2(x))}{2\sin(x)\cos(x)}$$

$$= \frac{\cos^2(x)}{\cos(x)\sin(x)} - \frac{\sin^2(x)}{\cos(x)\sin(x)}$$

$$= \frac{\cos(x)}{\sin(x)} - \frac{\sin(x)}{\cos(x)}$$

$$= \cot(x) - \tan(x)$$

Ex: $\sin(4x) = 4\sin(x)\cos^3(x) - 4\cos(x)\sin^3(x)$

Soln: Take $\sin(2\theta) = 2\cos(\theta)\sin(\theta)$

Take $\theta = 2x$

$$\sin(4x) = 2\cos(2x)\sin(2x)$$

Start w/ left:

$$\sin(4x) = 2\cos(2x)\sin(2x)$$

$$= 2\left(\cos^2(x) - \sin^2(x)\right)2\cos(x)\sin(x)$$

$$= 4\cos^3(x)\sin(x) - 4\sin^3(x)\cos(x)$$

Do it "WRONG"

$$\sin(4x) = 2\cos(2x)\sin(2x)$$

$$= 2(1 - 2\sin^2(x))(2\sin(x)\cos(x))$$

$$= 4\sin(x)\cos(x) - 8\sin^3(x)\cos(x)$$

STUCK

Start over