Double Angle identities

$$\sin(2\theta) = 2\cos(\theta)\sin(\theta)$$

$$\cos(2\theta) = \cos^2(\theta) - \sin^2(\theta)$$

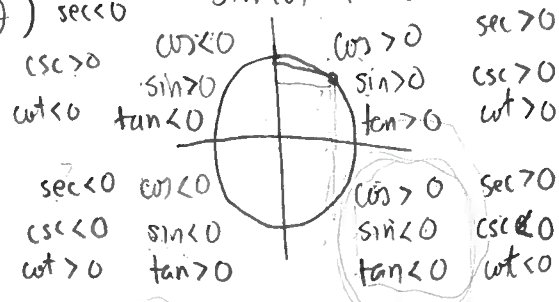
$$= 1 - 2\sin^2(\theta)$$

$$= 2\cos^2(\theta) - 1$$

$\cos^2(\theta) + \sin^2(\theta) = 1$

$\cos^2(\theta) = 1 - \sin^2(\theta)$

$\sin^2(\theta) = 1 - \cos^2(\theta)$



Ex: If $\tan(\theta) = -\frac{1}{9}$ and $\sin(\theta) < 0$, then

compute $\sin(2\theta)$, $\cos(2\theta)$, and $\tan(2\theta)$.

Soln: What Quadrant is θ in?

From "$\tan(\theta) = -\frac{1}{9}$" $\longrightarrow$ $\tan\theta < 0$

$\longrightarrow$ θ is in QII or QIV

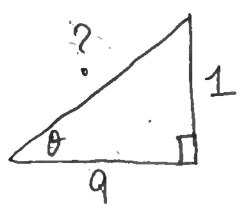
From "$\sin(\theta) < 0$" $\longrightarrow$ θ is in QIII or QIV

$\Downarrow$

Therefore, θ is in QIV.

From "$\tan\theta = \frac{-1}{9}$"

$\downarrow$



$\xrightarrow{\text{Pyth thm}}$ $9^2 + 1^2 = ?^2$

$\boxed{? = \pm\sqrt{82}}$

Compute

$$\sin(2\theta) = 2\underbrace{\cos(\theta)}_{>0}\underbrace{\sin(\theta)}_{<0} = \frac{2}{1}\left(\frac{9}{\sqrt{82}}\right)\left(-\frac{1}{\sqrt{82}}\right) = -\frac{18}{82}$$

$$= -\frac{9}{41}$$

b/c θ is in QIV

Now compute

$$\cos(2\theta) \underset{\substack{\uparrow \\ \text{dbl angle} \\ \text{identity}}}{=} \cos^2(\theta) - \sin^2(\theta) = \Big(\overset{\substack{\cos\theta \\ \downarrow}}{\frac{9}{\sqrt{82}}}\Big)^2 - \Big(\overset{\substack{\sin\theta \\ \downarrow}}{-\frac{1}{\sqrt{82}}}\Big)^2$$

$$= \frac{81}{82} - \frac{1}{82} = \frac{80}{82} = \frac{40}{41}$$

$$\tan(2\theta) = \frac{\sin(2\theta)}{\cos(2\theta)} = \frac{\left(-9/41\right)}{\left(40/41\right)} = \left(-\frac{9}{\cancel{41}}\right)\left(\frac{\cancel{41}}{40}\right) = -\frac{9}{40}$$

<u>Ex</u>: If $\sec(\theta) = -4$ and $\csc(\theta) < 0$, then find $\sin(2\theta)$, $\cos(2\theta)$, and $\tan(2\theta)$.

<u>Soln</u>: From "$\sec(\theta) = -4$" $\longrightarrow$ $\sec(\theta) < 0$

$\longrightarrow$ θ lies in QII or QIII

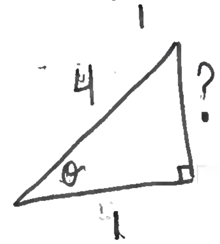
From "$\csc(\theta) < 0$" $\longrightarrow$ θ lies in QIII or QIV

$\downarrow$

θ lies in QIII

From "$\sec(\theta) = \frac{-4}{1}$",

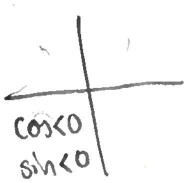
draw



$\xrightarrow{\text{Pyth thm}}$ $1^2 + ?^2 = 4^2$

$?^2 = 16 - 1 = 15$

$? = +\sqrt{15}$



Compute

$$\sin(2\theta) \overset{\substack{\text{dbl angle} \\ \text{ident} \\ \downarrow}}{=} 2\underbrace{\underbrace{\cos(\theta)}_{<0}\ \underbrace{\sin(\theta)}_{<0}}_{\theta \text{ in QIII}} = 2\left(-\frac{1}{4}\right)\left(-\frac{\sqrt{15}}{4}\right) = \frac{2\sqrt{15}}{16}$$

$$\cos(2\theta) \overset{\text{dbl ang}}{=} \cos^2(\theta) - \sin^2(\theta)$$

$$= \left(-\frac{1}{4}\right)^2 - \left(-\frac{\sqrt{15}}{4}\right)^2$$

$$= \frac{1}{16} - \frac{15}{16} = -\frac{14}{16} = -\frac{7}{8}$$

$$\tan(2\theta) = \frac{\sin(2\theta)}{\cos(2\theta)} = \frac{\left(\frac{2\sqrt{15}}{6}\right)}{\left(-7/8\right)} = -\left(\frac{2\sqrt{15}}{\cancel{6}_{3}}\right)\left(\frac{\cancel{8}^{4}}{7}\right) = -\frac{8\sqrt{15}}{21}$$

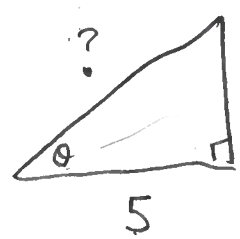
Ex: Simplify

$$\sin\left(2\arctan\left(\frac{x}{5}\right)\right)$$

Soln: Let $\theta = \arctan\left(\frac{x}{5}\right) \longrightarrow \tan\theta = \frac{x}{5}$.

θ is in QI or QIV

$\downarrow$ draw Δ



$\rightarrow 5^2 + x^2 = ?^2$

$\rightarrow ? = \sqrt{25+x^2}$

Compute

$x^2 = a$

$x = \pm\sqrt{a}$

$$\sin\left(2\arctan\left(\frac{x}{5}\right)\right) = \sin(2\theta)$$

$$\overset{\text{dbl angle}}{=} 2\sin(\theta)\cos(\theta)$$

$$= 2\left(\frac{x}{\sqrt{25+x^2}}\right)\left(\frac{5}{\sqrt{25+x^2}}\right)$$

$$= \frac{10x}{x^2+25}$$