Ex: Simplify

$$\frac{\sin(t)-1}{\cos(t)} + \frac{\cos(t)}{\sin(t)+1}$$

$\frac{1}{3} + \frac{1}{8} = \frac{8}{24} + \frac{3}{24}$

$= \frac{8+3}{24}$

Soln: Get a common denominator:

$$\frac{(\sin(t)-1)(\sin(t)+1)}{\cos(t)(\sin(t)+1)} + \frac{\cos(t)\cos(t)}{\cos(t)(\sin(t)+1)}$$

$\sin^2(t) - 1$

Pyth ident

$\cos^2(t) + \sin^2(t) = 1$

$$= \frac{(\sin^2(t) - 1) + \cos^2(t)}{\cos(t)(\sin(t)+1)}$$

$$= \frac{(\sin^2(t) + \cos^2(t)) - 1}{\cos(t)(\sin(t)+1)}$$

$$= \frac{1-1}{\cos(t)(\sin(t)+1)} = 0$$

this calculation shows

"the identity $\frac{\sin(t)-1}{\cos(t)} + \frac{\cos(t)}{\sin(t)+1} = 0$"

Proving identities

Rules of thumb

1. write everything in terms of sin and cos
2. anytime trig^2 ~ consider Pythagorean identity
3. Start on one side & arrive at other side

Ex: Prove identity

$$\cos(x) - \cos^3(x) = \cos(x)\sin^2(x)$$

Soln: Start w/ left:

$\cos^2(x) + \sin^2(x) = 1$

$\sin^2(x) = 1 - \cos^2(x)$

factor

$$\cos(x) - \cos^3(x) = \cos(x)\left(1 - \cos^2(x)\right)$$

$a(b+c) = ab + ac$

Pyth ident

$$= \cos(x)\sin^2(x)$$

QED

Ex: Prove identity

$$\frac{1+\tan^2(x)}{\csc^2(x)} + \sin^2(x) + \frac{1}{\sec^2(x)} = \sec^2(x)$$

Soln: Start w/ left:

Rule of thumb!

$$\frac{1+\tan^2(x)}{\csc^2(x)} + \sin^2(x) + \frac{1}{\sec^2(x)} = \frac{1+\frac{\sin^2(x)}{\cos^2(x)}}{\frac{1}{\sin^2(x)}} + \sin^2(x) + \frac{1}{\frac{1}{\cos^2(x)}}$$

$\frac{\frac{a}{b}}{\frac{c}{d}} = \left(\frac{a}{b}\right)\left(\frac{d}{c}\right)$

algebra

$$= \left(1+\frac{\sin^2(x)}{\cos^2(x)}\right)\left(\frac{\sin^2(x)}{1}\right) + \sin^2(x) + 1\cdot\left(\frac{\cos^2(x)}{1}\right)$$

got common denom

$$= \left(\frac{\cos^2(x)+\sin^2(x)}{\cos^2(x)}\right)\sin^2(x) + \frac{\sin^2(x)\cos^2(x)}{\cos^2(x)} + \frac{\cos^4(x)}{\cos^2(x)}$$

$= 1$ (Pythag ident)

add fractions →

$$= \frac{\sin^2(x) + \left[\sin^2(x)\cos^2(x) + \cos^4(x)\right]}{\cos^2(x)}$$

factor

$$= \frac{\sin^2(x) + \cos^2(x)\left[\sin^2(x) + \cos^2(x)\right]}{\cos^2(x)}$$

$\sin^2(x) + \cos^2(x) = 1$ Pythident

$$= \frac{\sin^2(x) + \cos^2(x)}{\cos^2(x)}$$

$\sin^2(x) + \cos^2(x) = 1$

$$= \frac{1}{\cos^2(x)} = \sec^2(x)$$

∎