Ex: Solve for A

$$\frac{\csc(t)-1}{\csc(t)+1} = \frac{A-\sin(t)}{A+\sin(t)}$$

Multiply both sides by $(\csc(t)+1)(A+\sin(t))$

Soln:

$$(\csc(t)-1)(A+\sin(t)) = (A-\sin(t))(\csc(t)+1)$$

trapped inside binomials

expand

$$(\csc(t)-1)A + (\csc(t)-1)(\sin(t)) = A\cdot(\csc(t)+1) - \sin(t)(\csc(t)+1)$$

$$A\csc(t) - A - A\csc(t) - A = \left(\frac{1}{\sin(t)} - 1\right)\sin(t) - \sin(t)\left(\frac{1}{\sin(t)}+1\right)$$

$$-2A = 1(-\sin(t)) - (1+\sin(t))$$

$$-1 - \sin(t)$$

$$-2A = -1+\sin(t) - 1 - \sin(t)$$

$$-2A = -2$$

$$\boxed{A = \frac{-2}{-2} = 1}$$

Ex: Simplify $\cot\left(\arcsin\left(\frac{x}{3}\right)\right)$

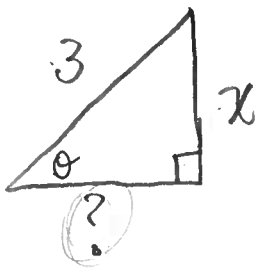
$\cot \sim \frac{\text{adj}}{\text{opp}}$

Soln: Let $\theta = \arcsin\left(\frac{x}{3}\right)$. Then $\sin(\theta) = \frac{x}{3}$.

using inverse property

θ in QI or QIV

Draw a Δ:



Pythm thm

$?^2 + x^2 = 3^2$

$? = \pm\sqrt{9-x^2}$

Therefore,

$\cot\left(\arcsin\left(\frac{x}{3}\right)\right) = \cot(\theta)$

$= \frac{\sqrt{9-x^2}}{x}$

tempting

$\frac{\sqrt{9-x^2}}{x} = \sqrt{\frac{9-x^2}{x^2}}$

$= \sqrt{\frac{9}{x^2} - 1}$

fails b/c x can be negative

Ex: Factor (write as a product)

$$\cot(x)\cos(x) - 5\cos(x) + 4\cot(x)$$

Common factors

Soln:

$$\left(\frac{\cos(x)}{\sin(x)}\right)\cos(x) - 5\cos(x) + 4\left(\frac{\cos(x)}{\sin(x)}\right)$$

$$= \cos(x)\left(\frac{\cos(x)}{\sin(x)} - 5 + \frac{4}{\sin(x)}\right)$$

$$= \cos(x)\left(\cot(x) - 5 + 4\csc(x)\right)$$

Ex: Factor

$$\cot(x)\cos(x) - 3\cos(x) + 4\cot(x) - 12$$

no longer a common factor among all terms

Soln:

$$\left(\frac{\cos(x)}{\sin(x)}\right)\cos(x) - 3\cos(x) + \frac{4\cos(x)}{\sin(x)} - 12$$

$$= \cos(x)\left(\frac{\cos(x)}{\sin(x)} - 3\right) + 4\left(\frac{\cos(x)}{\sin(x)} - 3\right)$$

common factor

$$= \left(\frac{\cos(x)}{\sin x} + 3\right)\left(\cos(x) + 4\right)$$

$$= \left(\cot(x) + 3\right)\left(\cos(x) + 4\right)$$

EX: Use identities to simplify

$$\frac{\cos^4(x) - \cos^2(x)}{\csc(x)}$$

Soln:

$$= \frac{\cos^2(x)\left(\cos^2(x) - 1\right)}{\frac{1}{\sin(x)}}$$

"flip + mult"

$$= \cos^2(x)\left(\cos^2(x) - 1\right)\left(\sin(x)\right)$$

$$= \cos^2(x)\left(-\sin^2(x)\right)\sin(x)$$

$$= -\cos^2(x)\sin^3(x)$$

$$x^4 - x^2 = x^2(x^2 - 1)$$

$$= \frac{\cos^2(x)\left(\cos^2(x) - 1\right)}{\frac{1}{\sin(x)}}\left(\frac{\sin(x)}{\sin(x)}\right), \quad \frac{\sin(x)}{\sin(x)} = 1$$

General rule

Anytime "trig^2" appears, consider Pythagorean identity.

$$\cos^2(x) + \sin^2(x) = 1$$

$$\cos^2(x) - 1 = -\sin^2(x)$$

$\tan^2$

Ex: Simplify

$$\frac{-\cot^2(x)-1}{\csc^2(x)}$$

Soln: $= \frac{-\csc^2(x)}{\csc^2(x)}$

$= -1$

$\cos^2(x)+\sin^2(x)=1$

↓ div both sides by $\cos^2(x)$

div by $\sin^2(x)$

$1+\tan^2(x)=\sec^2(x)$

$\cot^2(x)+1=\csc^2(x)$

↓ mult by (-1)

$-\cot^2(x)-1=-\csc^2(x)$