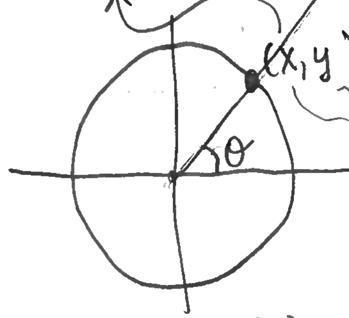
Pythagorean identit~~y~~ies

fact: "$x^2+y^2=1$" ⇝ defines unit circle



$(x,y)=(\cos\theta, \sin\theta)$

Since $x=\cos(\theta)$ and $y=\sin(\theta)$,

$$x^2+y^2=1$$

$$\downarrow$$

$$\boxed{\cos^2(\theta)+\sin^2(\theta)=1}$$

$$\frac{a+b}{c}=\frac{a}{c}+\frac{b}{c}$$

one of the most important trig identities

div by $\sin^2(\theta)$

$$\frac{\cos^2(\theta)+\sin^2(\theta)}{\sin^2(\theta)}=\frac{1}{\sin^2(\theta)}$$

$$\boxed{\cot^2(\theta)+1=\csc^2(\theta)}$$

div by $\cos^2(\theta)$

$$\frac{\cos^2(\theta)+\sin^2(\theta)}{\cos^2(\theta)}=\frac{1}{\cos^2(\theta)}$$

$$\boxed{1+\tan^2(\theta)=\sec^2(\theta)}$$

Ex: Simplify ← expand & write w/ fewest symbols possible

$(a+b+c)(d+e)$

$a(b+c) = ab + ac$

$$(\tan(\theta)+1)^2 = (\tan(\theta)+1)(\tan(\theta)+1)$$

$$= (\tan(\theta)+1)\tan(\theta) + (\tan(\theta)+1)(1)$$

$$= \tan^2(\theta) + \tan(\theta) + \tan(\theta) + 1$$

$$= \tan^2(\theta) + 2\tan(\theta) + 1$$

$\sec^2(\theta)$

$$= 2\tan(\theta) + \sec^2(\theta)$$

Ex: Simplify

$$\csc(\arccos(x))$$

$\tan^{-1} = \arctan$

$\cos^{-1} = \arccos$

Soln: Let $\boxed{\theta = \arccos(x)}$

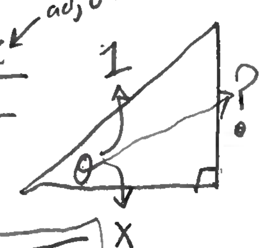
$\downarrow$ take cos

$$\cos(\theta) = \cancel{\cos(\arccos}(x)) = x$$

$$\cos(\theta) = x = \frac{x}{1}$$

adj θ

hyp



Pyth thm: $x^2 + ?^2 = 1^2$

$?^2 = 1 - x^2 \rightarrow \boxed{? = +\sqrt{1-x^2}}$

So, $\csc(\arccos(x)) = \csc(\theta) = \dfrac{1}{\sqrt{1-x^2}}$ ← only valid for $-1 \le x \le 1$

$\frac{x^{\cancel{2}}}{\cancel{x}}$

Ex: Simplify

$\cot(x)\sin(x)+\sec(x)\cos^2(x)$

Proof: Rewrite in terms of sin + cos

$\frac{a}{b}\cdot\frac{c}{d}=\frac{ac}{bd}$

$$\cot(x)\sin(x)+\sec(x)\cos^2(x) = \left(\frac{\cos(x)}{\cancel{\sin(x)}}\right)\cancel{\sin(x)}+\left(\frac{1}{\cos(x)}\right)\cos^2(x)$$

ansatz → "assumed form"

$$\left(\frac{1}{\cos(x)}\right)\cdot\left(\frac{\cos^2(x)}{1}\right)$$

$$=\frac{\cos^2(x)}{\cos(x)}=\frac{\cos(x)\cos(x)}{\cos(x)}$$

$$=\frac{\cos(x)}{1}\cdot\frac{\cos(x)}{\cos(x)}=1$$

$$=\cos(x)$$

$$=\cos(x)+\cos(x)$$

$$=2\cos(x)$$

Recall factoring

$$x^2+bx+c=(x+p)(x+q)$$

$$=x^2+(p+q)x+pq$$

$$\Rightarrow\begin{cases}b=p+q\\ c=pq\end{cases}$$

$p=-2,\ q=1\rightarrow p+q=-1$

$p=2,\ q=-1\rightarrow p+q=1$

$b=1$

$c=-2$

Ex:

$$\frac{\cos^2(x)+\cos(x)-1}{2\cos(x)-2}=\frac{w^2+w-2}{2w-2}$$

"quadratic in form"

introduce

$\hookrightarrow w=\cos(x)$

$w^2=\cos^2(x)$

$$=\frac{(w+2)\cancel{(w-1)}}{2\cancel{(w-1)}}$$

$$=\frac{w+2}{2}$$

$$=\frac{\cos(x)+2}{2}$$

④

Ex: Simplify

$$\cot(-t)\sec(-t)\sin(t) = \frac{\cos(-t)}{\sin(-t)}\,\frac{1}{\cos(-t)}\,\sin(t)$$

even+odd

$$= \frac{\cos(t)}{-\sin(\cancel{t})}\cdot\frac{1}{\cancel{\cos}(t)}\,\sin(\cancel{t})$$

$$= -1$$

$$-\frac{\cos(t)}{\cos(t)}\,\frac{\sin(t)}{\sin(t)}$$

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$$\frac{a}{b} + \frac{c}{d} = \frac{e+f}{g+h}$$

t t t

Ex: Solve

$$\frac{\csc(t)-1}{\csc(t)+1} = \frac{A-\sin(t)}{A+\sin(t)}$$

Soln: Mult by $(A+\sin(t))(\csc(t)+1)$

$$(\csc(t)-1)(A+\sin(t)) = (A-\sin(t))(\csc(t)+1)$$

MONDAY