Ex: Compute

$$\sec\Big(\underbrace{\arcsin\left(\tfrac{1}{3}\right)}_{\text{is an angle in QI}}\Big)$$

$\cancel{\text{trig}(\text{arctrig})}$

arctrig(trig) ~ careful

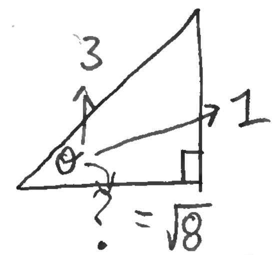
Soln: Let $\theta = \arcsin\left(\frac{1}{3}\right)$.

$\downarrow$ take sin of both sides

$\cancel{\tan(\theta) = \frac{1}{3}}$

$\cancel{\frac{\sin\theta}{\cos\theta} = \frac{1}{3}}$

$$\sin(\theta) = \frac{1}{3} \longrightarrow$$



$\downarrow$ Pyth thm

$$?^2 + 1^2 = 3^2$$

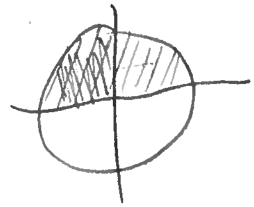
$$?^2 = 9 - 1 = 8$$

$$? = \pm\sqrt{8}$$

Now, can calculate

$$\sec\left(\arcsin\left(\tfrac{1}{3}\right)\right) = \sec(\theta) = \frac{3}{\sqrt{8}}$$

Ex: Compute $\csc\big(\underbrace{\arccos(-\tfrac{1}{4})}_{\text{angle in QII}}\big)$



and $\sec\big(\arccos(-\tfrac{1}{4})\big)$.

Soln: Let $\theta = \arccos(-\tfrac{1}{4})$

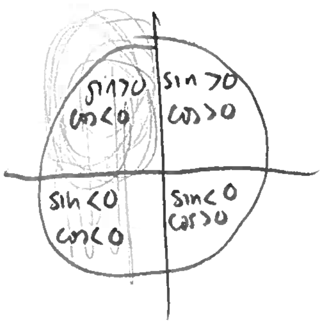
$\big\downarrow$ take cos

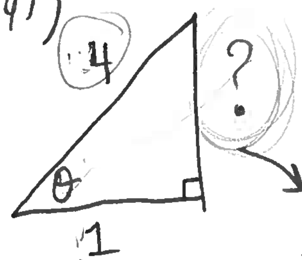
$\cos\theta = \cancel{\cos}(\cancel{\arccos}(-\tfrac{1}{4}))$

$\cos(\theta) = -\tfrac{1}{4}$

$\uparrow$

θ must be in QII or QIII





$1^2 + ?^2 = 4^2$

$?^2 = 15$

$? = \pm\sqrt{15}$

Compute

in QII ⇒ sin > 0 ⇒ $\csc = \frac{1}{\sin} > 0$ (+)

$\csc\big(\arccos(-\tfrac{1}{4})\big) = \csc(\theta)$

$= +\dfrac{4}{\sqrt{15}}$

in QII → $\cos < 0$ → $\sec = \frac{1}{\cos} < 0$

$\sec\big(\arccos(-\tfrac{1}{4})\big) = \sec(\theta)$

$= -4$

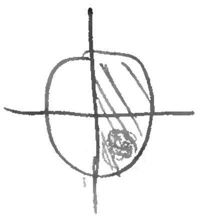
annihilate

Ex: $\cancel{\tan}\left(\cancel{\arctan}\left(-\frac{2}{3}\right)\right) = -\frac{2}{3}$

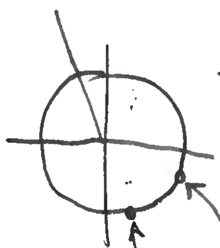
"Which angle in QI or QIV has a tangent = 1?"

$$\cancel{\arctan}\left(\cancel{\tan}\left(\frac{\pi}{4}\right)\right) = \arctan\left(\frac{\sqrt{2}/2}{\sqrt{2}/2}\right) = \arctan(1)$$

$$= \frac{\pi}{4}$$



$$\arctan\left(\tan\left(\frac{2\pi}{3}\right)\right) = \arctan\left(\frac{\sqrt{3}/2}{-1/2}\right)$$



$$\frac{\frac{a}{b}}{\frac{c}{d}} = \frac{a}{b}\cdot\frac{d}{c}$$

$$= \arctan\left(\frac{\sqrt{3}}{\cancel{2}}\left(-\frac{\cancel{2}}{1}\right)\right)$$

$$= \arctan(-\sqrt{3})$$

"Which angle in QI or QIV has a tangent = $-\sqrt{3}$?"

$$= -\frac{\pi}{3}$$

$\theta = -\frac{\pi}{6}$

$(\sqrt{3}/2, -1/2)$

$$\tan\left(-\frac{\pi}{6}\right) = \frac{-1/2}{\sqrt{3}/2}$$

$$= \left(-\frac{1}{\cancel{2}}\right)\left(\frac{\cancel{2}}{\sqrt{3}}\right)$$

$$= -\frac{1}{\sqrt{3}}$$

$\theta = -\frac{\pi}{3}$

$(\frac{1}{2}, -\sqrt{3}/2)$

$$\tan\left(-\frac{\pi}{3}\right)$$

$$= \frac{-\sqrt{3}/2}{1/2}$$

$$= -\frac{\sqrt{3}}{\cancel{2}}\cdot\frac{\cancel{2}}{1} = -\sqrt{3}$$

Ex: $\arcsin(0) = 0$

"Which angle in QI or QIV has a sine = 0?"

$\arccos\left(-\frac{\sqrt{3}}{2}\right) = \frac{5\pi}{6}$

"Which angle in QI or QII has a cosine = $-\frac{\sqrt{3}}{2}$?"

Trigonometric identities

Equation

$2x^2 = 8$

Goal: usually to "find x" because the equation isn't ALWAYS TRUE

$x=1$: $2(1^2) = 8$
$2 = 8$
false!

$x=2$: $2 \cdot 2^2 = 8$
$2 \cdot 4 = 8$
true!

↓

trig eqts

↓

find all values of angle

Identity

$3x + x = 4x$

Goal: usually to "prove" it because the equation here is ALWAYS TRUE

$x=1$: $3+1=4$ ✓ TRUE

$x=2$: $3(2)+2 = 4(2)$
$6+2=8$
true!

↓

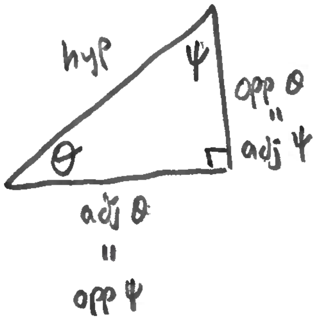
trig identities

↓

establish the thing is true using known information (usually other identities)

Identities we have seen

cofunction identities



$$\sin(\theta) = \frac{\text{opp } \theta}{\text{hyp}} = \frac{\text{adj } \psi}{\text{hyp}} = \cos(\psi)$$

<u>FACT</u>: $\theta + \psi + \frac{\pi}{2} = \pi$

right angle

$$\Downarrow$$

$$\theta + \psi = \frac{\pi}{2}$$

$$\boxed{\psi = \frac{\pi}{2} - \theta}$$

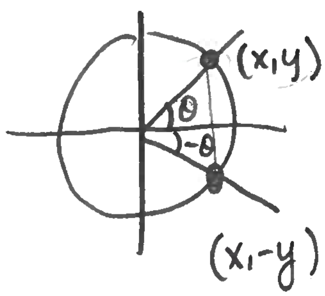
$\boxed{\sin(\theta) = \cos\left(\frac{\pi}{2} - \theta\right)}$	$\cos(\theta) = \sin\left(\frac{\pi}{2} - \theta\right)$
$\tan(\theta) = \cot\left(\frac{\pi}{2} - \theta\right)$	$\cot(\theta) = \tan\left(\frac{\pi}{2} - \theta\right)$
$\sec(\theta) = \csc\left(\frac{\pi}{2} - \theta\right)$	$\csc(\theta) = \sec\left(\frac{\pi}{2} - \theta\right)$

cofunction identities

Definitional identities

$\tan(\theta) = \dfrac{\sin\theta}{\cos\theta}$ $\qquad \csc(\theta) = \dfrac{1}{\sin\theta}$ $\qquad \sec(\theta) = \dfrac{1}{\cos(\theta)}$

Even + odd



$\cos(-\theta) = x = \cos(\theta) \leftarrow$ "cosine is an even function"

$\sin(-\theta) = -y = -\sin(\theta) \leftarrow$ "sine is an odd function"

interesting connection

Can show:

$5! = 5\cdot 4\cdot 3\cdot 2\cdot 1$

powers are odd

$$\sin(x) = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \dots$$

powers are even

$$\cos(x) = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \dots$$

"infinite polynomial"

~ "power series" — calc 2 topic

$(-x)^2 = x^2$

$= (-x)(-x)$

$(-x)^3 = -x^3$

$= (-x)(+x)(+x)$