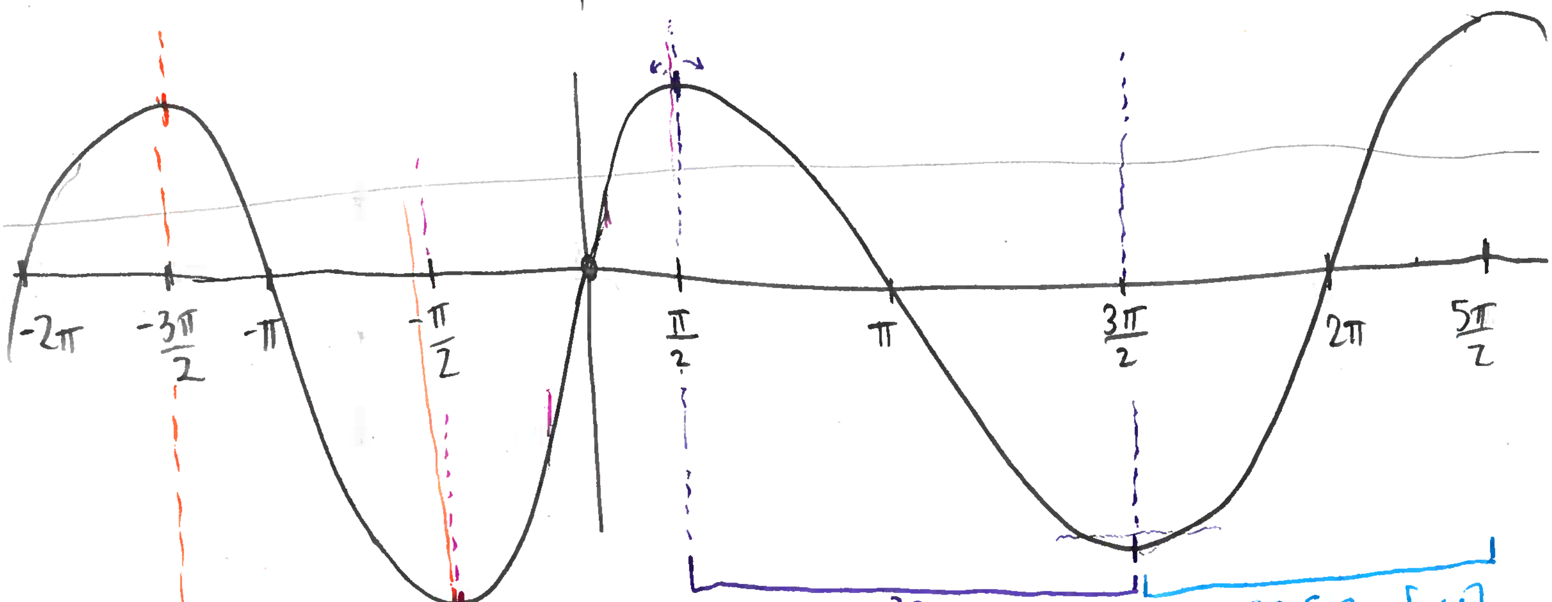
THE PROBLEM: sine, cosine, tangent are not one-to-one

$$\left.\begin{aligned} \sin &: \mathbb{R} \to [-1,1] \\ \cos &: \mathbb{R} \to [-1,1] \\ \tan &: \underbrace{\mathbb{R}}_{\text{domain}} \to \underbrace{\mathbb{R}}_{\substack{\text{codomain} \\ \text{(here: range)}}} \end{aligned}\right\} \text{not one-to-one}$$

Need to do → domain restriction to get one-to-one functions which will have inverses



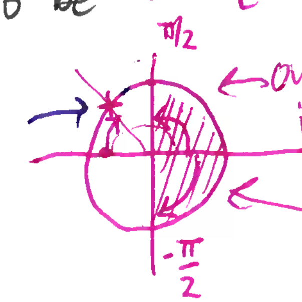
$\sin: \left[-\frac{3\pi}{2}, -\frac{\pi}{2}\right] \to [-1,1]$ ↓ is one-to-one

$\sin: \left[-\frac{\pi}{2}, \frac{\pi}{2}\right] \to [-1,1]$ is one-to-one ⇓ has an inverse

$\sin: \left[\frac{\pi}{2}, \frac{3\pi}{2}\right] \to [-1,1]$ is one-to-one ⇓ would have an inverse

$\sin: \left[\frac{3\pi}{2}, \frac{5\pi}{2}\right] \to [-1,1]$ is one-to-one

All these options are valid BUT most common (pretty much all calculators) is to take the domain restriction to be $\sin: \left[-\frac{\pi}{2}, \frac{\pi}{2}\right] \to [-1,1]$



← output of inverse sine lives here in QI or QIV

Using restriction $\sin : \left[-\frac{\pi}{2}, \frac{\pi}{2}\right] \to [-1,1]$

we obtain inverse sine $\sin^{-1}$

$$\arcsin : [-1,1] \to \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$$

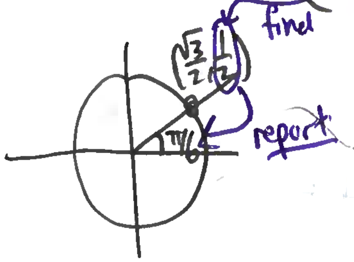
inputs — where outputs live

EX: What is $\arcsin\left(\frac{1}{2}\right)$?

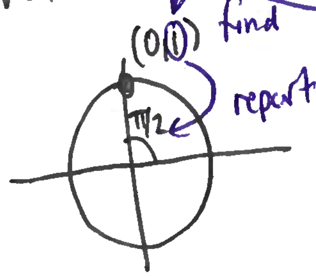
Soln: "What angle in QI or QIV has a sine equal to $\frac{1}{2}$?"

$$\arcsin\left(\frac{1}{2}\right) = \frac{\pi}{6}$$

read from unit circle



EX: What is $\arcsin(1)$?



$$\arcsin(1) = \frac{\pi}{2}$$

Ex: Compute $\sin(\arcsin(1/2))$

$\cancel{f}(\cancel{f}^{-1}(x)) = x$

$\cancel{f^{-1}}(\cancel{f}(x)) = x$

Soln: Since sin and arcsin are inverses, they annihilate + we get

$$\cancel{\sin}(\cancel{\arcsin}(1/2)) = \frac{1}{2}$$

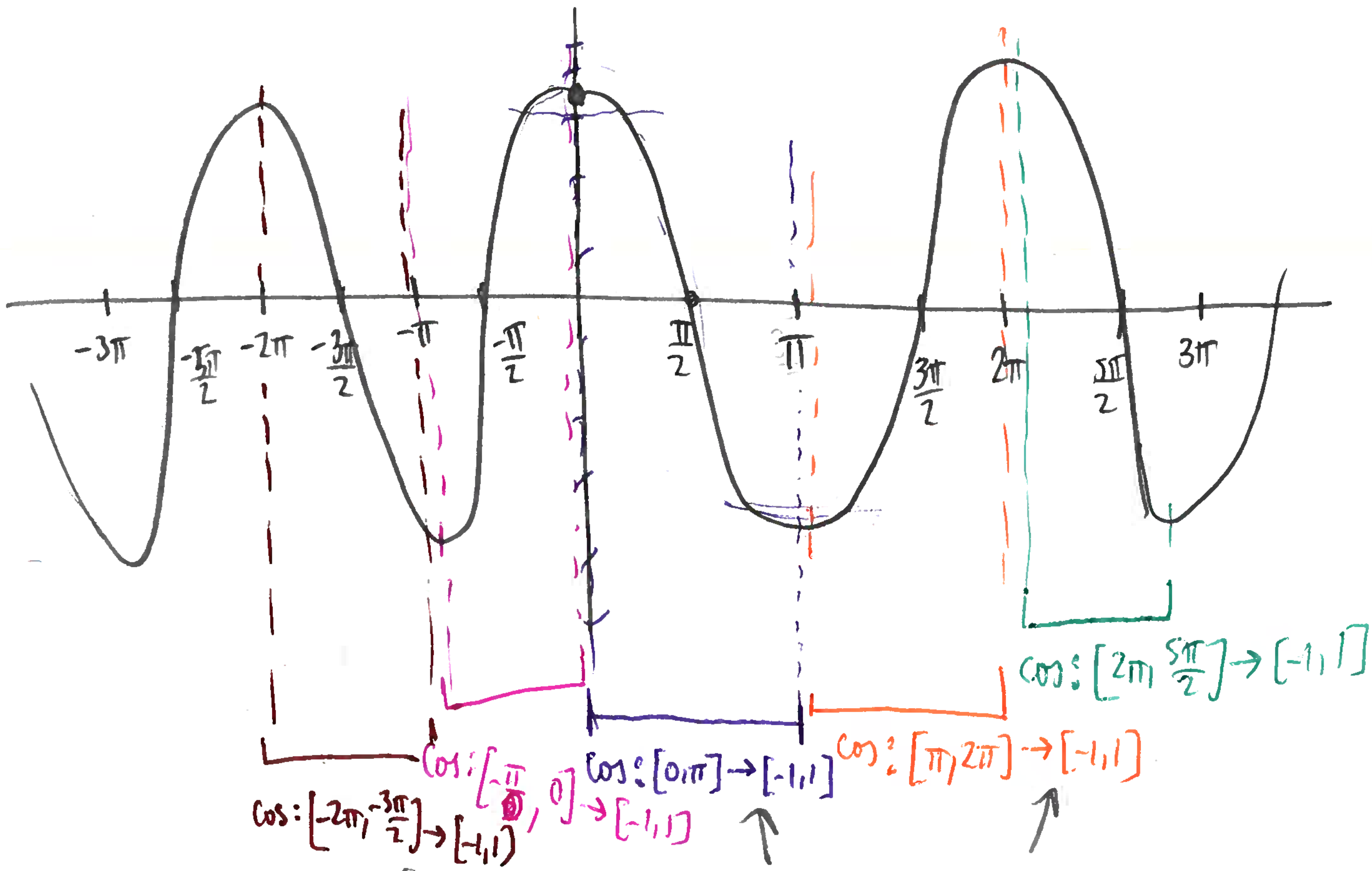
Ex: $\cancel{\arcsin}(\cancel{\sin}(\pi/4)) = \frac{\pi}{4}$

Ex: $\underbrace{\arcsin\left(\boxed{\sin\left(\frac{3\pi}{4}\right)}\right)}_{\text{can't just annihilate}} = \arcsin\left(\frac{\sqrt{2}}{2}\right) = \frac{\pi}{4}$

Because $\frac{3\pi}{4}$ is in Q II but arcsin outputs to Q I or Q IV

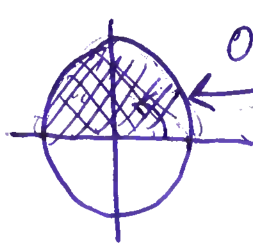
MORAL: always make sure that output of any arcsin lies in appropriate quadrant

Look at arccos



all of these domain restrictions give one-to-one functions

Most common: cos: [0, π] → [-1, 1]



outputs of arccos generated from this lie in QI or QII

We define $\arccos: [-1,1] \to [0,\pi]$

$$\underset{\text{inputs}}{[-1,1]} \to \underset{\text{where outputs live}}{[0,\pi]}$$

<u>EX</u>: What is...

$\arccos\left(\frac{1}{2}\right) =$ "what angle in QI or QII has cosine of $1/2$?"

$= \frac{\pi}{3}$

<u>Ex</u>: What is

$$\cancel{\cos}\left(\cancel{\arccos}\left(\frac{\sqrt{3}}{2}\right)\right) = \cos\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2}$$

in general:

$$\cancel{\text{trig}}(\cancel{\text{arctrig}}(x)) = x \quad \text{(always)}$$

$\text{arctrig}(\text{trig}(x)) \to$ <u>BE CAREFUL</u>

<u>Ex</u>: $\arccos\left(\cos\left(-\frac{\pi}{2}\right)\right) = \arccos(0)$

$= \frac{\pi}{2}$