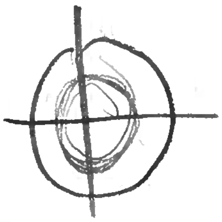
Ex: Compute $\sin\left(\frac{25\pi}{3}\right)$ ~~using ref ∠'s~~



Soln: Realize that $\frac{25\pi}{3}$ is more than one full rotation — because 2π is one full rotation & $\frac{25}{3} > 2$. So we should "unwind" that: we have to subtract full rotations ~~of~~ $2\pi = \frac{6\pi}{3}$ until we reach a small enough value (within one full rotation).

$$\frac{25\pi}{3} - \underset{\substack{\uparrow \\ \text{1 rotation}}}{\frac{6\pi}{3}} = \frac{19\pi}{3}$$

$$\frac{19\pi}{3} - \frac{6\pi}{3} = \frac{13\pi}{3}$$

$$\frac{13\pi}{3} - \frac{6\pi}{3} = \frac{7\pi}{3}$$

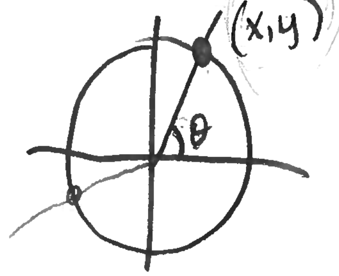
$$\frac{7\pi}{3} - \frac{6\pi}{3} = \frac{\pi}{3} \checkmark$$

Therefore,

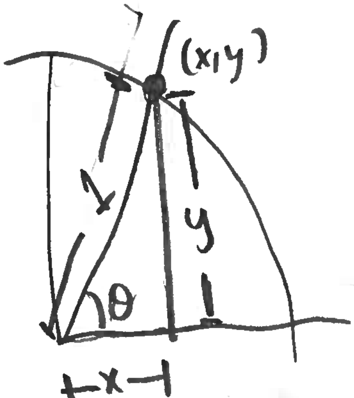
$$\sin\left(\frac{25\pi}{3}\right) = \sin\left(\frac{\pi}{3}\right) = \frac{\sqrt{3}}{2}$$

Other 4 trig functions

We know



$$\Rightarrow \begin{cases} \cos(\theta) = x \\ \sin(\theta) = y \end{cases}$$



$$\sin(\theta) = \frac{\text{opp } \theta}{\text{hyp}} = \frac{y}{1}$$

$$\tan\theta = \frac{\text{opp}}{\text{adj}}$$

$$\frac{\sin\theta}{\cos\theta} = \frac{\text{opp}/\text{hyp}}{\text{adj}/\text{hyp}}$$

$$= \frac{\text{opp}}{\cancel{\text{hyp}}} \cdot \frac{\cancel{\text{hyp}}}{\text{adj}}$$

$$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{y}{x}$$

$$\cot\theta = \frac{\cos\theta}{\sin\theta} = \frac{x}{y}$$

$$\sec\theta = \frac{1}{\cos\theta} = \frac{1}{x}$$

$$\csc\theta = \frac{1}{\sin\theta} = \frac{1}{y}$$

$$\frac{1}{\tan(\theta)} = \cot(\theta)$$

$$\frac{1}{\cot(\theta)} = \tan(\theta)$$

$$\frac{1}{\cos\theta} = \sec\theta$$

$$\frac{1}{\sec\theta} = \cos\theta$$

$$\frac{1}{\csc\theta} = \sin\theta$$

Ex: Find all 6 trig functions at $\theta = \frac{5\pi}{3}$.

direct from unit circle

$$\cos(\theta) = \frac{1}{2}$$
$$\sin(\theta) = -\sqrt{3}/2$$
$$\tan(\theta) = \frac{-\sqrt{3}/2}{1/2} = -\frac{\sqrt{3}}{\cancel{2}} \cdot \frac{\cancel{2}}{1} = -\sqrt{3}$$
$$\cot(\theta) = \frac{1}{\tan\theta} = -\frac{1}{\sqrt{3}}$$
$$\sec(\theta) = 2$$
$$\csc(\theta) = -\frac{2}{\sqrt{3}}$$

ref angle

$\theta = \frac{5\pi}{3}$ is in QIV

$$\text{ref}_\angle = 2\pi - \frac{5\pi}{3} = \frac{6\pi}{3} - \frac{5\pi}{3} = \frac{\pi}{3}$$

$$\cos(\theta) = \oplus\cos(\text{ref}_\angle) = \cos\left(\frac{\pi}{3}\right) = \frac{1}{2}$$

QIV

$$\sin(\theta) = \ominus\sin\left(\frac{\pi}{3}\right) = -\frac{\sqrt{3}}{2}$$

$$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{-\sqrt{3}/2}{1/2} = -\sqrt{3}$$

etc...

Ex: Think about signs of trig functions:

$\sin\theta > 0$ $\cos\theta < 0$ $\tan\theta < 0$	$\cos\theta > 0$ $\sin\theta > 0$ $\tan\theta > 0$
$\cos\theta < 0$ $\sin\theta < 0$ $\tan\theta > 0$	$\cos\theta > 0$ $\sin\theta < 0$ $\tan\theta < 0$

Ex: Convert $\frac{\pi}{8}$ rad to degrees.

2π rad $=$ 1 full rot $= 360^\circ \longrightarrow 1 = \frac{360^\circ}{2\pi \text{ rad}}$

$$\frac{\pi}{8}\text{ rad} = \left(\frac{\cancel{\pi}}{8}\cancel{\text{rad}}\right)\left(\frac{360^\circ}{2\cancel{\pi}\,\cancel{\text{rad}}}\right) = \left(\frac{360}{16}\right)^\circ = \frac{180}{8}^\circ = \frac{90}{4}^\circ = \left(\frac{45}{2}\right)^\circ$$

$$= 22.5^\circ$$

Ex: Compute

$$\tan(-60^\circ) = \frac{-\sqrt{3}/2}{1/2} = -\sqrt{3}$$

$$\csc(-60^\circ) = \frac{1}{1/2} = 2$$

~~Ex~~ Pythagorean identity

Unit circle obeys $x^2+y^2=1$

$\frac{a+b}{c} = \frac{a}{c}+\frac{b}{c}$

BUT we know $\cos\theta = x$

$\sin\theta = y$

SO $\boxed{\cos^2(\theta)+\sin^2(\theta)=1}$ ← memorize!

divide both sides by $\cos^2(\theta)$

$$\frac{1}{\cos^2(\theta)} = \frac{\cos^2(\theta)+\sin^2(\theta)}{\cos^2(\theta)}$$

$$\sec^2(\theta) = \frac{\cos^2(\theta)}{\cos^2(\theta)} + \frac{\sin^2(\theta)}{\cos^2(\theta)}$$

$$\boxed{\sec^2(\theta) = 1+\tan^2(\theta)}$$

divide both sides by $\sin^2(\theta)$

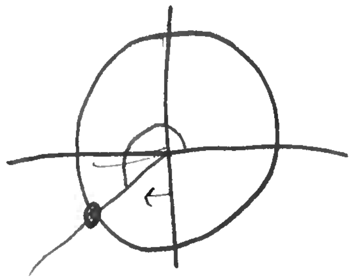
$$\frac{\cos^2(\theta)+\sin^2(\theta)}{\sin^2(\theta)} = \frac{1}{\sin^2(\theta)}$$

$$\frac{\cos^2(\theta)}{\sin^2(\theta)} + \frac{\sin^2(\theta)}{\sin^2(\theta)} = \csc^2(\theta)$$

$$\boxed{\cot^2(\theta)+1 = \csc^2(\theta)}$$

<u>Ex</u>: Given that $\sin(\theta) = \frac{-1}{6}$ and θ is in QIII, find all other trig functions.

<u>Soln</u>:



$\Rightarrow \cos(\theta) = -\frac{\sqrt{35}}{6}$

etc....

<u>Using eqt of circle</u>

$$x^2 + \left(-\frac{1}{6}\right)^2 = 1$$

$$x^2 = 1 - \frac{1}{36} = \frac{35}{36}$$

$$x = \pm\sqrt{\frac{35}{36}} = \pm\frac{\sqrt{35}}{6}$$

QIII

$$x = -\frac{\sqrt{35}}{6}$$

<u>Using Pythagorean identity</u>

$$\cos^2(\theta) + \sin^2(\theta) = 1$$

$$\cos^2(\theta) + \left(-\frac{1}{6}\right)^2 = 1$$

$$\cos^2(\theta) = \frac{35}{36}$$

$$\cos(\theta) = \pm\frac{\sqrt{35}}{6}$$

$\big\downarrow$ QIII

$$\cos(\theta) = -\frac{\sqrt{35}}{6}$$

$$\Rightarrow \tan(\theta) = \frac{\sin\theta}{\cos\theta} = \frac{-1/6}{-\sqrt{35}/6} = \frac{1}{\sqrt{35}}$$

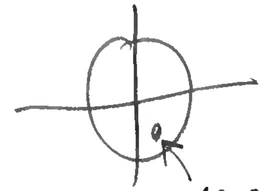
$$\cot(\theta) = \sqrt{35}$$

$$\sec(\theta) = -\frac{6}{\sqrt{35}}$$

$$\csc(\theta) = -6$$

Ex: Given $\tan(\theta) = -32$ & θ is in Q IV.

$\frac{64}{2}$ $\frac{16}{1/2}$



$\cos\theta > 0$

$\boxed{\frac{1}{\cos\theta} = \sec\theta > 0}$

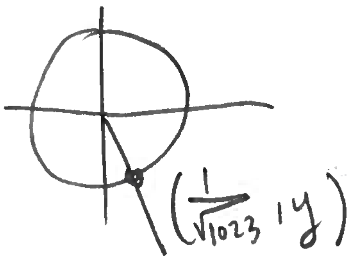
$$\sec^2(\theta) + 1 = (-32)^2$$

$$\sec^2(\theta) = 1023$$

$$\sec(\theta) = \pm\sqrt{1023}$$

$\frac{1}{\sec\theta} = \cos(\theta)$

$\downarrow$ Q IV



$$\sec(\theta) = \pm\sqrt{1023}$$

$$\cos(\theta) = \frac{1}{\sqrt{1023}}$$

$$\left(\frac{1}{\sqrt{1023}}\right)^2 + \sin^2(\theta) = 1$$

$$\sin^2(\theta) = \frac{1023}{1023} - \frac{1}{1023} = \frac{1022}{1023}$$

$$\sin\theta = \pm\sqrt{\frac{1022}{1023}} \longleftrightarrow \sin(\theta) = -\sqrt{\frac{1022}{1023}}$$

$$\cot(\theta) = \frac{1}{\tan\theta} = -\frac{1}{32}$$

$$\csc\theta = \frac{1}{\sin\theta} = -\sqrt{\frac{1023}{1022}}$$