x

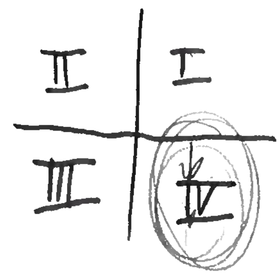
Ex: If $\left(\frac{5}{6}, y\right)$ is on unit circle, is in QIV, then find y.

eqt defining unit circle

Soln: We substitute $x = \frac{5}{6}$ into $x^2 + y^2 = 1$.

$$\left(\frac{5}{6}\right)^2 + y^2 = 1$$

$$\frac{25}{36} + y^2 = \frac{36}{36}$$



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TRIG

$$y^2 = \frac{36-25}{36} = \frac{11}{36}$$

$$\Downarrow$$

$$y = \pm\sqrt{\frac{11}{36}} = \pm\frac{\sqrt{11}}{6}$$

$\Rightarrow$ because of QIV, $y = -\frac{\sqrt{11}}{6}$

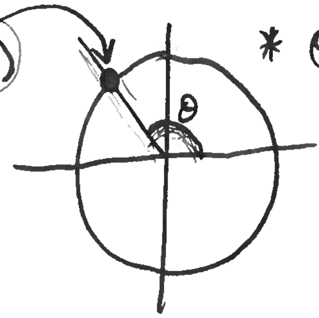
$\Rightarrow \left(\frac{5}{6}, -\frac{\sqrt{11}}{6}\right)$

Trig functions on unit circle

We extend the ideas of right-Δ trig to all angles via the unit circle:

(x, y)

lies on unit circle (must obey $x^2+y^2=1$)



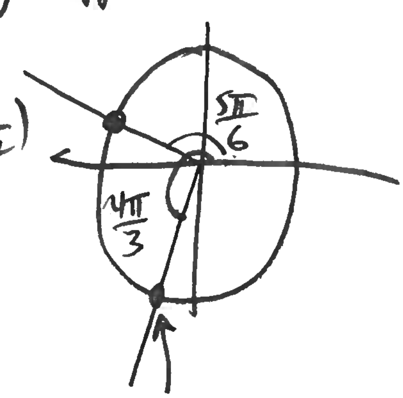
* θ is an angle

$x = \cos(\theta)$

$y = \sin(\theta)$

This means that you can compute a trig function of any angle on unit circle by reading off the point that the angle cuts the circle at.

$\left(-\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$



<u>Ex</u>: Compute

$(\cos(\theta), \sin(\theta)) = (x, y)$

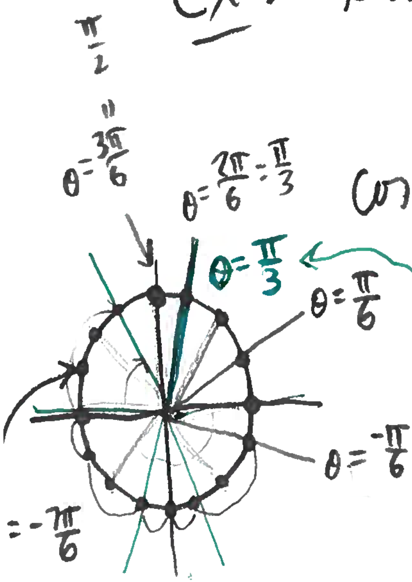
$$\cos\left(\frac{5\pi}{6}\right) = -\frac{\sqrt{3}}{2}$$

$$\sin\left(\frac{4\pi}{3}\right) = -\frac{\sqrt{3}}{2}$$

$\left(-\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$

$$\cos\left(\frac{4\pi}{3}\right) = -\frac{1}{2}$$

Ex: $\sin\left(-\frac{7\pi}{6}\right) = \frac{1}{2}$



$\cos\left(\frac{9\pi}{3}\right) = \frac{1}{2}$

also $\frac{7\pi}{3}$

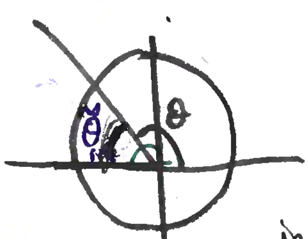
$\left(-\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$

$\sin\left(\frac{10\pi}{4}\right) =$ 2nd coord of $(0,1)$ $= 1$

Reference angle - another way to make sense of angles that arise from existing angles on unit circle

A ref angle is the angle in QI whose coordinates are most similar to a given angle.

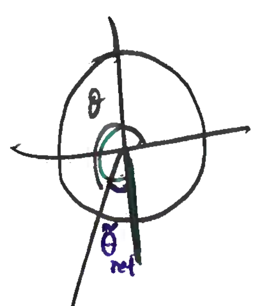
if θ in QII



$\theta_{ref} = 180^\circ - \theta = \underbrace{\pi - \theta}_{\text{in radians}}$

↑ in QI ~ its measure is $< 90^\circ$

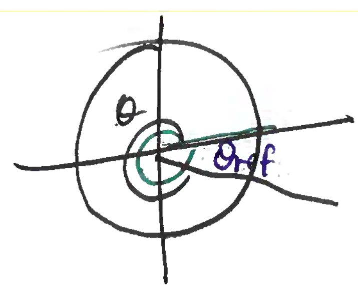
if θ is in QIII



$\theta_{ref} = 270^\circ - \theta$

$= \frac{3\pi}{2} - \theta$

if θ in QIV

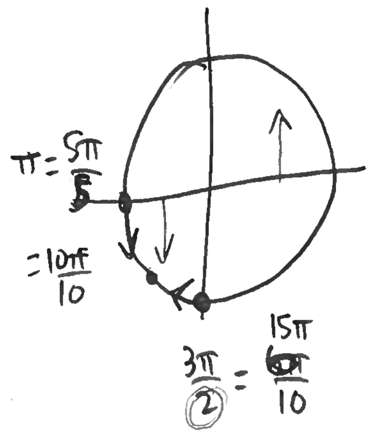


$\theta_{ref} = 360^\circ - \theta$

$= 2\pi - \theta$

Ex: Find ref ∠ for $\theta = \frac{7\pi}{5}$.

$\frac{14\pi}{10}$



Here, θ is in QIII b/c

$$\underbrace{\pi = \frac{10\pi}{10}}_{\text{edge of QII - QIII}} < \underbrace{\frac{7\pi}{5} = \frac{14\pi}{10}}_{\text{is in QIII}} < \underbrace{\frac{3\pi}{2} = \frac{15\pi}{10}}_{\text{edge of QIII to QIV}}$$

Therefore, ref ∠ of $\frac{7\pi}{5}$ is

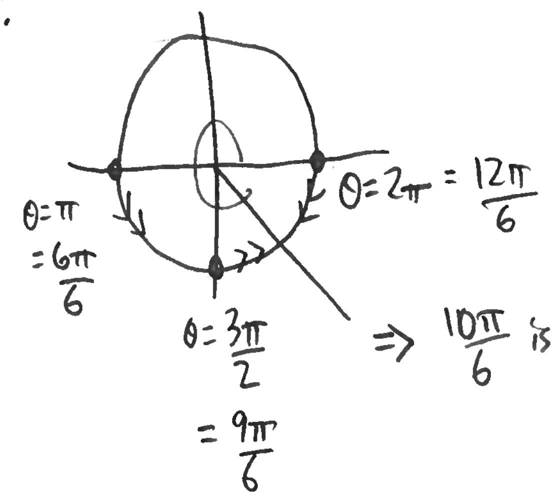
$$\text{ref}\angle = \frac{3\pi}{2} - \frac{7\pi}{5} = \frac{15\pi}{10} - \frac{14\pi}{10} = \frac{\pi}{10}$$

So for ex,

$$\boxed{\sin\left(\frac{7\pi}{5}\right) = -\sin\left(\frac{\pi}{10}\right)}$$

Ex: Use ref ∠ to find $\cos\left(\frac{10\pi}{6}\right)$

Soln:



$\Rightarrow \frac{10\pi}{6}$ is in QIV

So,

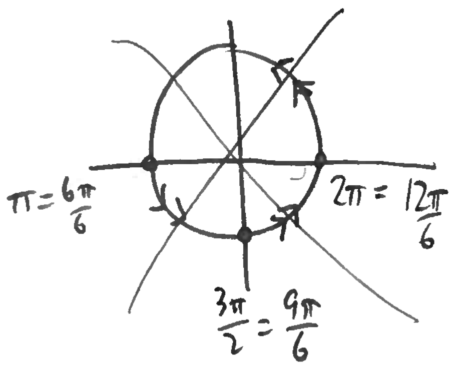
$$\text{ref}\angle = 2\pi - \frac{10\pi}{6} = \frac{12\pi}{6} - \frac{10\pi}{6} = \frac{2\pi}{6} = \frac{\pi}{3}$$

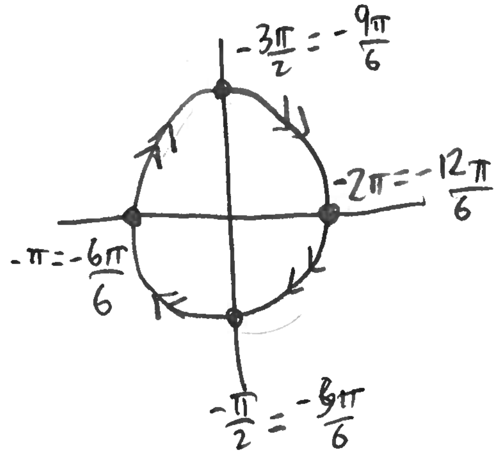
Since cosine > 0 in QIV,

$$\cos\left(\frac{10\pi}{6}\right) = +\cos\left(\frac{\pi}{3}\right) = \frac{1}{2}$$

Ex: Compute $\sin\left(-\frac{32\pi}{6}\right)$ using ref $\angle$.

Soln: Find quadrant $-\frac{32\pi}{6}$ is in:



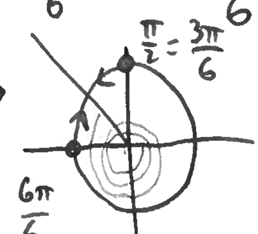


"Unwind" extra rotations:

$$-\frac{32\pi}{6} + 2\pi = -\frac{32\pi}{6} + \frac{12\pi}{6} = -\frac{20\pi}{6}$$

$$-\frac{20\pi}{6} + 2\pi = -\frac{20\pi}{6} + \frac{12\pi}{6} = -\frac{8\pi}{6}$$

$$-\frac{8\pi}{6} + \frac{12\pi}{6} = \frac{4\pi}{6}$$



these describe same position on unit CIRCLE

in QII $\Rightarrow$ ref $\angle = \pi - \frac{4\pi}{6} = \frac{2\pi}{6} = \frac{\pi}{3}$

$$\Rightarrow \sin\left(-\frac{32\pi}{6}\right) \underset{\text{unwind}}{=} +\sin\left(\frac{4\pi}{6}\right) \underset{\text{ref}\angle}{=} +\sin\left(\frac{\pi}{3}\right) = \frac{\sqrt{3}}{2}$$