

Law of Cosines

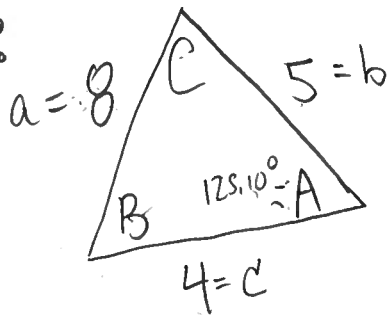
1

$$a^2 = b^2 + c^2 - 2bc \cos(A)$$

$$b^2 = a^2 + c^2 - 2ac \cos(B)$$

$$c^2 = a^2 + b^2 - 2ab \cos(C)$$

Ex:



Find A

$$8^2 = 5^2 + 4^2 - 2(5)(4) \cos(A)$$

$$64 - 25 - 16 = -40 \cos(A)$$

$$\frac{23}{-40} = \cos(A)$$

$$A = \arccos\left(\frac{23}{-40}\right) \approx 125.10^\circ$$

plug into
arccos

$$\arccos\left(\frac{23}{-40}\right)$$

"

$$\arccos(\cos(A))$$

Find B

$$5^2 = 4^2 + 8^2 - 2(4)(8) \cos(B)$$

$$\frac{25 - 16 - 64}{-64} = \cos(B)$$

$$B = \arccos\left(\frac{-55}{-64}\right) \approx 30.75^\circ$$

alternatively

$$16 = 64 + 25 - 2(8)(5) \cos(C)$$

$$C = \arccos\left(\frac{16 - 64 - 25}{-80}\right)$$

$$\approx 24.15^\circ$$

Find C

$$125.10^\circ + 30.75^\circ + C = 180^\circ$$

$$C = 180^\circ - 125.10^\circ - 30.75^\circ$$

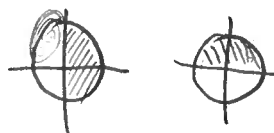
$$= 24.15^\circ$$

if you use arcsin:

MUST check for 2nd soln

if you use arccos:

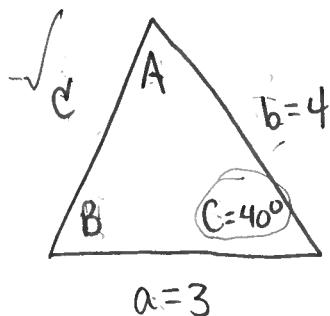
NOT check for 2nd soln



Ex: Solve Δ given by $a=3, b=4, C=40^\circ$

(2)

Soln:



Find c

$$c^2 = 3^2 + 4^2 - 2(3)(4)\cos(40^\circ)$$

\downarrow

$$c = \pm \sqrt{9 + 16 - 24\cos(40^\circ)}$$

\downarrow

$$c = \oplus \sqrt{\quad} \approx 2.57$$

Find A

$$3^2 = 4^2 + (2.57)^2 - 2(4)(2.57)\cos(A)$$

$$\cos(A) = \frac{9 - 16 - (2.57)^2}{-2(4)(2.57)}$$

$$A = \arccos(\quad) \approx 48.57^\circ$$

Find B

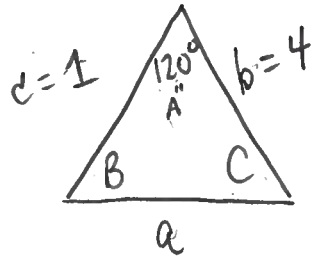
$$40^\circ + 48.57^\circ + B = 180^\circ$$

$$B = 180^\circ - 40^\circ - 48.57^\circ \approx 91.43^\circ$$

R

Ex: Solve $b=4, c=1, A=120^\circ$

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Soln:

Find a

$$a^2 = 4^2 + 1^2 - 2(4)(1)\cos(120^\circ)$$

↓

$$a = \pm \sqrt{4^2 + 1^2 - 2(4)\cos(120^\circ)}$$

↓

$$a = +\sqrt{16 + 1 - 8\cos(120^\circ)} \approx 4.58$$

Find C

$$1^2 = (4.58)^2 + 4^2 - 2(4.58)(4)\cos(C)$$

$$\cos(C) = \frac{1 - (4.58)^2 - 16}{-2(4.58)(4)}$$

$$C = \arccos(\text{''}) \approx 10.92^\circ$$

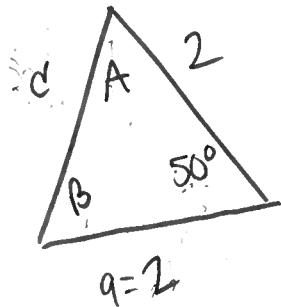
Find B

$$120^\circ + B + 10.92^\circ = 180^\circ$$

$$B = 180^\circ - 120^\circ - 10.92^\circ = 49.08^\circ$$

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EX:



Soln: Find c

$$c^2 = 2^2 + 2^2 - 2(2)(2)\cos(50^\circ)$$

$$c = \sqrt{8 - 8\cos(50^\circ)} \approx 1.69$$

Alternative

$$\frac{\sin(50^\circ)}{1.69} = \frac{\sin(A)}{2}$$

$$A = \arcsin\left(\frac{2\sin(50^\circ)}{1.69}\right)$$

$$\approx 65.03^\circ$$

2nd soln?

$$A_2 = 180^\circ - 65.03^\circ$$

$$\approx 114.97^\circ$$

↓

$$114.97^\circ + 50^\circ + B = 180^\circ$$

$$B = 15.03^\circ$$

NOT A Δ — "sanity check" fails

— smallest angle is $B = 15.03^\circ$

BUT smallest side is $c = 1.69$

Find A

$$2^2 = 2^2 + (1.69)^2 - 2(2)(1.69)\cos(A)$$

$$A = \arccos\left(\frac{2^2 - 2^2 - (1.69)^2}{-2(2)(1.69)}\right) \approx 65.01^\circ$$

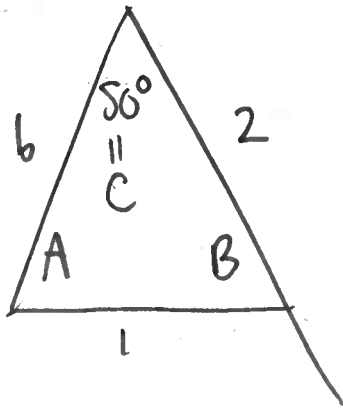
Find B

$$65.01^\circ + B + 50^\circ = 180^\circ$$

$$B = 180^\circ - 65.01^\circ - 50^\circ = 64.99^\circ$$

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Why can't I always use law of cosines?



$$2^2 = b^2 + 1^2 - 2b(1)\cos(A) \quad \times$$

$$b^2 = 2^2 + 1^2 - 2(2)(1)\cos(B) \quad \times$$

~~Q3~~

$$1^2 = 2^2 + b^2 - 2b\cos(50^\circ)$$

$$b^2 - 2b\cos(50^\circ) + 3 = 0$$

↓ QF

$$b = \frac{2\cos(50^\circ) \pm \sqrt{(2\cos(50^\circ))^2 - 4(1)(3)}}{2}$$