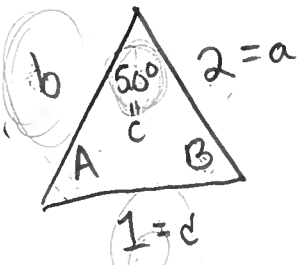
Law of sines

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

Ex: Solve



Soln: Find A

$$\frac{\sin(A)}{2} = \frac{\sin(50^\circ)}{1}$$

$$\sin(A) = 2\sin(50^\circ)$$

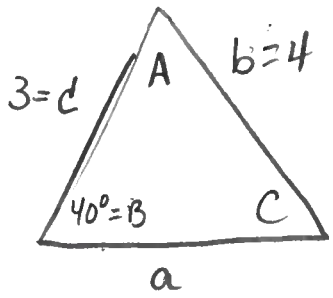
$$A = \arcsin(2\sin(50^\circ))$$

$\approx$ DOES NOT EXIST

$\Downarrow$

TRIANGLE is NOT SOLVABLE

Ex: Solve Δ $c=3$, $B=40^\circ$, $b=4$. $\Rightarrow$



Find C

$$\frac{\sin(C)}{3} = \frac{\sin(40^\circ)}{4}$$

$$\sin(C) = \frac{3\sin(40^\circ)}{4}$$

$$C = \arcsin\left(\frac{3\sin(40^\circ)}{4}\right)$$

$\approx 28.82^\circ$

check 2nd poss soln $\Rightarrow$

Find C_2

$C_2 = 180^\circ - 28.82^\circ = 151.18^\circ$

$\downarrow$

Find A

$40^\circ + 28.82^\circ + A = 180^\circ$

$A = 180^\circ - 40^\circ - 28.82^\circ$

$= 111.18^\circ$

Find a

$$\frac{\sin(111.18^\circ)}{a} = \frac{\sin(40^\circ)}{4}$$

$$a = \frac{4\sin(111.18^\circ)}{\sin(40^\circ)}$$

$$\approx 5.80$$

↓

Find A_2

$40^\circ + 151.18^\circ + A_2 = 180^\circ$

$A_2 = -11.18^\circ$

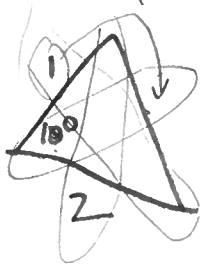
↑ makes no sense in Δ-trig !!

Summary

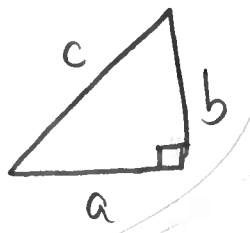
① given 2 angles + one side → law of sines but never 2 solns (b/c you don't use arcsin)

② if arcsin is ever used in a law of sines → must check for 2nd soln (happens when 1 angle + 2 sides and are able to use law of sines)

This Δ cannot be solved w/ law of sines:



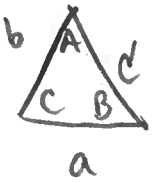
Pyth thm

if 

, then $a^2+b^2=c^2$

Law of cosines – generalization of Pythagorean theorem

* works for all Δ's
* becomes Pyth thm if Δ is a right Δ

if 

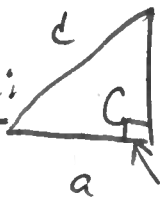
, then...

(1) $a^2 = b^2 + c^2 - 2bc\cos(A)$

(2) $b^2 = a^2 + c^2 - 2ac\cos(B)$

(3) $c^2 = a^2 + b^2 - 2ab\cos(C)$

FACT: $\cos(90°) = 0$

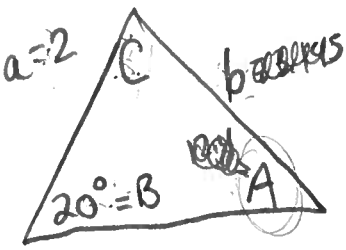
Consequence: 

LoC $\Rightarrow$ $c^2 = a^2 + b^2 - \cancel{2ab\cos(90°)}$

Pyth thm

BENEFIT of LoC: never a second solution!!

Ex: Solve



← cannot solve w/ law of sines

⇓

must use LoC

$\sqrt{c=5}$

Find b

Soln: We will use

$$b^2 = a^2 + c^2 - 2ac\cos(B)$$

⇓ plug in info

$$b^2 = \underbrace{2^2 + 5^2 - 2(2)(5)\cos(20^\circ)}_{\text{a number}} \approx 10.21$$

⇓ take $\sqrt{}$

$$b = \pm\sqrt{10.21} \rightarrow \boxed{b = 3.195}$$

Find C use $c^2 = a^2 + b^2 - 2ab\cos(C)$

⇓

$$5^2 = 2^2 + (3.195)^2 - 2(2)(3.195)\cos(C)$$

⇓ ALGEBRA

$$5^2 - 2^2 - (3.195)^2 = \underbrace{-2(2)(3.195)}_{\text{number}}\cos(C)$$

⇓

$$\cos(C) = \frac{21 - (3.195)^2}{-2(2)(3.195)} \Rightarrow C = \arccos\left(\frac{21-(3.195)^2}{-2(2)(3.195)}\right)$$

$$\approx 147.6^\circ$$

Find A

$$20^\circ + 147.6^\circ + A = 180^\circ$$

$$A = 180^\circ - 20^\circ - 147.6^\circ$$

$$= 12.4^\circ$$