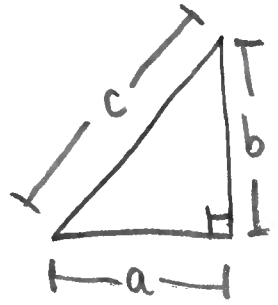
Pythagorean theorem

"$a^2+b^2=c^2$"

↑ not true for all Δ's

For a right Δ:

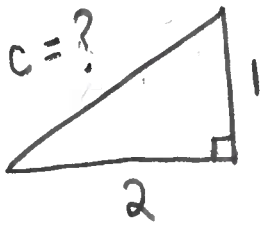


$a^2+b^2=c^2$

$4<5$

$2=\sqrt{4}<\sqrt{5}$

Ex: Solve the Δ:



Soln: $2^2+1^2=c^2$

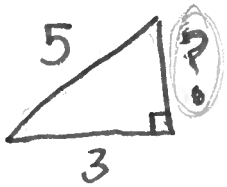
$4+1=c^2$

$5=c^2$

throw away negative soln

$c=\pm\sqrt{5} \Rightarrow \boxed{c=\sqrt{5}}$

Ex: Solve

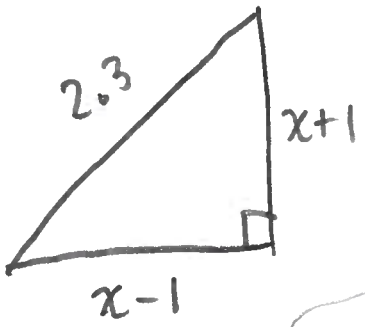


$\rightarrow$ $3^2+(?)^2=5^2$

$?^2=25-9=16$

$?=\pm\sqrt{16}=\pm 4 \rightarrow \boxed{?=4}$

EX: Solve



$$(x-1)^2 + (x+1)^2 = (2.3)^2$$

$$(x^2 - 2x + 1) + (x^2 + 2x + 1) = 5.29$$

$$2x^2 + 2 = 5.29$$

$$2x^2 = 3.29$$

$$x^2 = \frac{3.29}{2} = 1.645$$

$$x = \pm\sqrt{1.645} \longrightarrow x = \sqrt{1.645} \approx 1.283$$

$a(b \pm c) = ab \pm ac$ (a, b, c)

$$(x-1)^2 = (x-1)(x-1)$$

$$= (x-1)x + (x-1)(-1)$$

$$= x^2 - x - x - (-1)$$

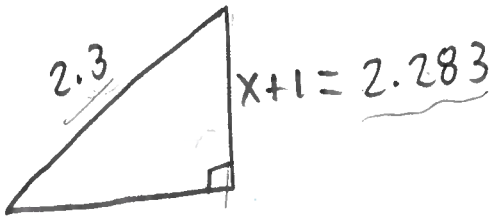
$$= x^2 - 2x + 1$$

$$(x+1)^2 = (x+1)(x+1)$$

$$= (x+1)x + (x+1)(1)$$

$$= x^2 + x + x + 1$$

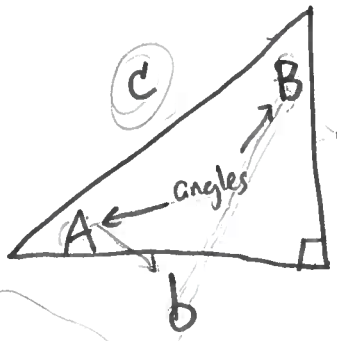
$$= x^2 + 2x + 1$$



Right-Δ Trigonometry

take an angle as an input

Six trigonometric functions



"sine" → $\sin(A) = \frac{a}{c} = \frac{\text{"opposite of A"}}{\text{hypotenuse}}$

"cosine" → $\cos(A) = \frac{b}{c} = \frac{\text{"adjacent to A"}}{\text{hypotenuse}}$

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"tangent" → $\tan(A) = \frac{a}{b} = \frac{\text{"opposite of A"}}{\text{"adjacent to A"}}$

FACT: $\frac{\sin(A)}{\cos(A)} = \frac{a/c}{b/c} = \left(\frac{a}{c}\right)\left(\frac{c}{b}\right) = \frac{a \cdot \not{c}}{b \cdot \not{c}} = \frac{a}{b} = \tan(A)$

"cosecant" → $\csc(A) = \frac{c}{a} = \frac{\text{hypotenuse}}{\text{"opposite of A"}}$

"secant" → $\sec(A) = \frac{c}{b} = \frac{\text{hypotenuse}}{\text{"adjacent to A"}}$

"cotangent" → $\cot(A) = \frac{b}{a} = \frac{\text{"adjacent to A"}}{\text{"opposite of A"}}$

$\cos(B) = \frac{a}{c} = \sin(A)$ $\quad \tan(B) = \frac{b}{a} = \cot(A)$

$\csc(B) = \frac{c}{b} = \sec(A)$

FACT: A triagle has 180° ~ $A + B + 90^\circ = 180^\circ$

→ $A + B = 90^\circ$

"A and B are complementary angles"

$A = 90^\circ - B$ $\quad B = 90^\circ - A$

EX: Spz we have a right Δ with $a=2$ and $c=6$.

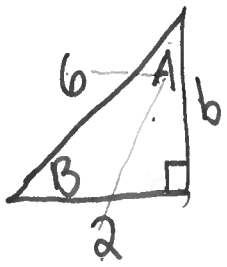
Find

$\sin(A)$

$\cos(A)$

$\tan(A)$

Soln:



b

$\sin(A) = \frac{2}{6} = \frac{1}{3}$

$\cos(A) = \frac{b}{6}$ $\quad \tan(A) = \frac{2}{b}$

we need b

Pyth thm: $2^2 + b^2 = 6^2$

$4 + b^2 = 6^2$

$b^2 = 36 - 4 = 32$

$b = \pm\sqrt{32} \longrightarrow \boxed{b = \sqrt{32}}$

~~$\sqrt{8}$~~

~~$2\sqrt{2}$~~

$\Rightarrow \cos(A) = \frac{\sqrt{32}}{6}$

$\tan(A) = \frac{2}{\sqrt{32}}$