Written HW1 → due today for timely completion

(1)

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"Course content" ~ WHW1 ~ upload there

First online HW — due Wed for timely completion

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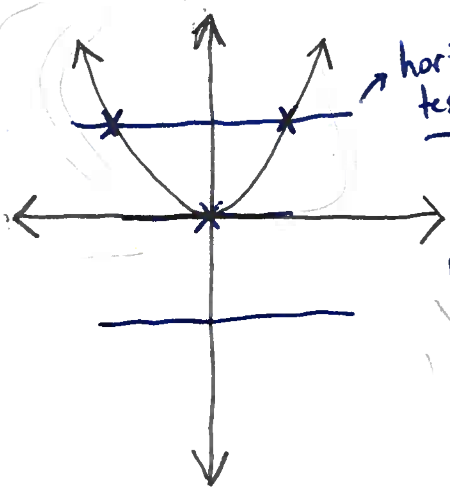
x-axis y-axis

Ex: $\begin{cases} f: \mathbb{R} \to \mathbb{R} \\ f(x) = x^2 \end{cases}$

Ex: $\begin{cases} g: [0,1] \to \mathbb{R} \\ g(x) = x^2 \end{cases}$

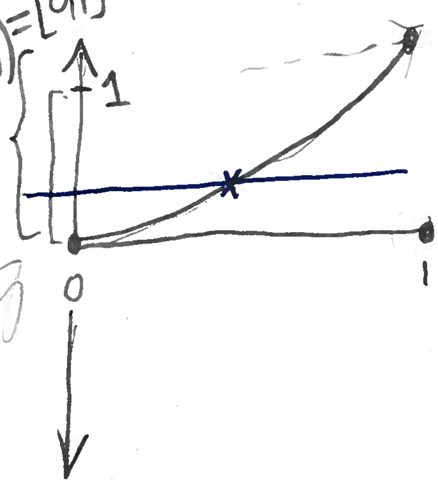


horiz line test fails

⇓

no inverse

range(g) = [0,1]

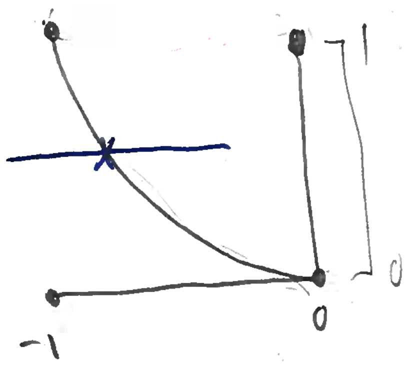


passes horiz line test

⇓

there is an inv.

Ex: $\begin{cases} h: [-1,0] \to [0,1] \\ h(x) = x^2 \end{cases}$



passes horiz line test

⇓

there is an inv

One-to-one functions

Obey "horizontal line test" —

"Does there exist a horiz line that touches more than one pt of the graph?"

yes → fail

NO → pass

Theorem: If a function passes the horizontal line test, then it has an inverse function. If it fails, then there is no inverse.

From previous page: g and h have inverses

<u>Recall</u>: if $\begin{cases} f: X \to Y \\ f(x) = \sim \end{cases}$ has an inverse, then

the inverse is a function

$$\begin{cases} f^{-1}: \text{range}(f) \to X \\ f^{-1}(x) \text{ obeys } \begin{array}{l} f^{-1}(f(x)) = x \\ f(f^{-1}(x)) = x \end{array} \end{cases}$$

~~inv f~~

~~f inv~~

<u>Ex</u>: $x^2 = 4$

$\downarrow$

$x = \pm\sqrt{4} = \pm 2$

<u>Ex</u>: $(\sqrt{x})^2 = x$

<u>Ex</u>: $e^{\ln(x)} = x$

$\ln(e^x) = x$

Look at g^{-1}

$$\begin{cases} g^{-1}: \underset{\substack{\uparrow \\ \text{range}(g)}}{[0,1]} \to [0,1] \\ g^{-1}(x) \text{ obeys } \begin{array}{l} g^{-1}(g(x)) = x \\ g(g^{-1}(x)) = x \end{array} \end{cases}$$

$\Downarrow$ TURNS OUT

$g^{-1}(x) = \sqrt{x}$

<u>Check</u>

$g^{-1}(g(x)) = g^{-1}(x^2)$

$= \sqrt{x^2}$

$= x$

$g(g^{-1}(x)) = g(\sqrt{x}) = (\sqrt{x})^2 = x$

Look at h^{-1}

$$\begin{cases} h^{-1}: [0,1] \to [-1,0] \\ h^{-1} \text{ obeys } \begin{array}{l} h^{-1}(h(x)) = x \\ h(h^{-1}(x)) = x \end{array} \end{cases}$$

$\Downarrow$ TURNS OUT

$h^{-1}(x) = -\sqrt{x}$

"$\sqrt{\ }$"

$\uparrow$

always $\oplus$

<u>Check</u>

$h^{-1}(h(x)) = h^{-1}(x^2) = -\sqrt{x^2}$

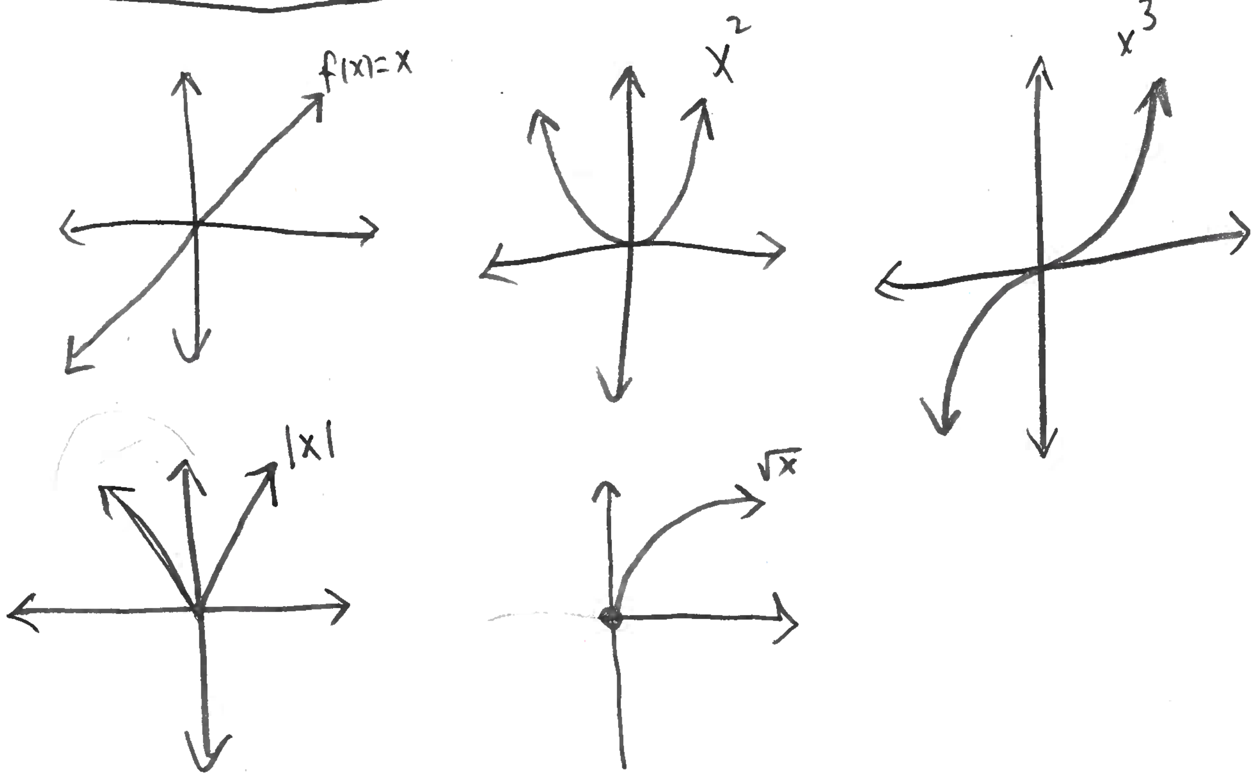
$= -x$

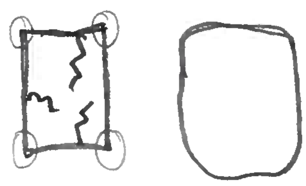
$h(h^{-1}(x)) = h(-\sqrt{x}) = (-\sqrt{x})^2$

$= (-\sqrt{x})(-\sqrt{x})$

$= x$

Library of functions





Function transformations

Ways to move a graph around + "mold" it

Assume c constant, $c>0$.

Formula	Fancy name	Actual action
$f(x) \pm C$	vertical shift	add/subtract to y-values
$f(x \pm c)$	horizontal shift	subtract/add to x-values
$\pm C f(x)$	vertical stretch or compress	multiply y-vals by $\pm C$
$f(\pm Cx)$	horizontal stretch/compress	divide x-vals by $\pm C$

EX: Plot $f(x) = 3(x-2)^2$

- "fundamental" function x^2
- the "-2" is a hor shift $\longrightarrow$ add 2 to x-vals
- the "3" is a vert stretch $\longrightarrow$ mult y-vals by 3