Carbon ~ usually has 6 protons and 6 neutrons

isotope ~ carbon-14

Ex: Human hair from an ancient body was found & it contains 75% of carbon 14 that occurs in natural tissue.

How old is the body? (note: half life of C-14 is 5730yr).

Soln: We model using

$y_0 e^{-rt}$ ← measured in years

↑ initial amount of C-14

$\ln\left(\frac{1}{2}\right) = \ln(1) - \ln(2)$, $\ln(1) = 0$

From half-life: $\frac{1}{2}y_0 = y_0 e^{5730r}$

$-\ln(2) = 5730r$

$r = \frac{-\ln(2)}{5730} \approx -0.000121$

Update model: $y_0 e^{-0.000121t}$

75% of initial amount $y_0 \sim 0.75 y_0$

$\Rightarrow 0.75y_0 = y_0 e^{-0.000121t}$

$\ln(0.75) = -0.000121t \rightarrow t = \frac{\ln(0.75)}{-0.000121} \approx 2377.54$

EX: Suppose growth rate of USA remains 0.66% per year for the coming time.

If current pop is 328 million, then how long will it take at this rate to reach 500 million? 1 billion? (1000 million)

Soln: $y_0(1+P)^t$ ← time period; t ← years

y_0: initial pop; P: percent growth

$0.66\% = \frac{0.66}{100} = 0.0066$

$$328(1+0.0066)^t$$

Q: How long until 500 million?

$$500 = 328(1.0066)^t \longrightarrow \frac{500}{328} = (1.0066)^t$$

$$\ln\left(\frac{500}{328}\right) = t\ln(1.0066)$$

$$t = \frac{\ln(500/328)}{\ln(1.0066)} \approx 64.09$$

Q: To 1 billion?

$$1000 = 328(1.0066)^t \longrightarrow t = \frac{\ln(1000/328)}{\ln(1.0066)} \approx 169.46$$