

"continuous" growth/decay \sim $y_0 e^{rt}$

y_0 initial amount
 r growth/decay rate

①

$(r > 0 \sim \text{growth})$
 $r < 0 \sim \text{decay}$

"percent growth/decay" \sim $y_0 (1 \pm p)^{\frac{t}{T}}$

y_0 initial amount
 t time period
 p percent growth/decay ($+ \text{growth}$, $- \text{decay}$)

Ex: A pop. of 300 fish ^{are} in a lake. Their pop decreases

starting by $\frac{1}{3}$ every 5 years.

$33.3\% \approx p = 0.333$
 decrease $\rightarrow -$

Express decay of this population in form

Recall $5^{-1} = \frac{1}{5}$

$= a b^{\frac{-t}{T}}$

$\frac{300}{(0.667)^{\frac{t}{5}}} = 300 \left(\frac{1}{0.667}\right)^{-\frac{t}{5}}$

$1 - 0.333 = 0.667$

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Ex: Currently 13 frogs in pond
 — Frog pop grows exponentially, doubles every 3 days

How long (in days) does it take for there to

be 200 frogs in pond?

2000 frogs in pond?

Sohl:

$$y_0(1+P)^{\frac{t}{T}}$$

$$= 13 \cdot (2^{\frac{t}{3}})$$

$$P=1$$

100% increase
every 3 days

$$\% = \frac{1}{100}$$

$$100\% = \frac{100}{100} = 1$$

Notice: if $t=3$:

$$13 \cdot 2^{\frac{3}{3}} = 13 \cdot 2 = 26$$

To get to 200:

$$\underbrace{200}_{\text{target we want to know}} = 13 \cdot 2^{\frac{t}{3}} \rightarrow \frac{200}{13} = 2^{\frac{t}{3}}$$

$$\ln\left(\frac{200}{13}\right) = \ln\left(2^{\frac{t}{3}}\right) \\ = \frac{t}{3} \ln(2)$$

$$\Rightarrow t = \frac{3 \ln\left(\frac{200}{13}\right)}{\ln(2)} \approx \boxed{11.83 \text{ days}}$$

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to 2000?

$$2000 = 13 \cdot 2^{\frac{t}{3}}$$

$$\ln\left(\frac{2000}{13}\right) = \frac{t}{3} \ln(2)$$

$$t = \frac{3 \ln\left(\frac{2000}{13}\right)}{\ln(2)} \approx 21.80 \text{ days}$$

Ex: A pop. 5.6 million grows at 1.1% per year.

What is the population in one century? ↑

100yr

T=1

$$\text{Solv: } 1.1\% = \frac{1.1}{100} = 0.011$$

Growth formula: t ← measured in years

$$\underbrace{5.6(1+0.011)}_{\text{millions of people}} = 5.6(1.011)^t$$

pop in 1 century: set $t=100$

$$5.6(1.011)^{100} \approx 16.72 \text{ million}$$

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Ex: Find doubling time of a city

whose pop grows 11% per year

Sohm:

$$P = 11\%$$

$$T = 1$$

$$= \frac{11}{100}$$

$$t \sim \text{years}$$

$$= 0.11$$

Growth model

$$y_0(1+0.11)^t = y_0(1.11)^t$$

Doubling time

$$2y_0 = y_0(1.11)^t$$

$$2 = (1.11)^t$$

$$\ln(2) = t \ln(1.11)$$

$$t = \frac{\ln(2)}{\ln(1.11)} \approx 6.64 \text{ years}$$

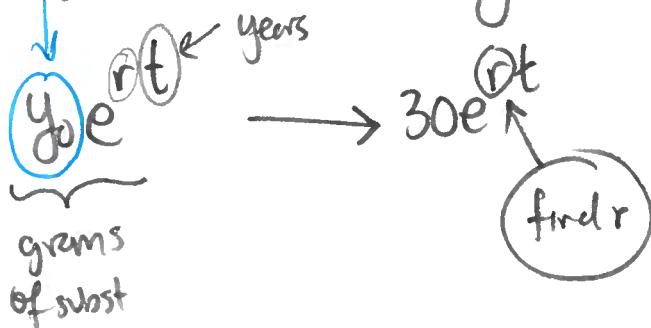
(5)

Ex: Half-life of a radioactive substance
is 10 yr.



If you have 30 grams of the substance
then how long will it take to get
only 1 gram?

Soln: Model as continuous decay:



B/c half life is 10 yr:

$$15 = 30 e^{10r}$$

↑
(I have 15 grams
at 10 year mark
(half-life))

$$\frac{1}{2} = \frac{15}{30} = e^{10r}$$

$$\ln\left(\frac{1}{2}\right) = \ln(e^{10r})$$

$$\ln(1) - \ln(2)$$

$$-\ln(2) = 10r$$

0

$$r = -\frac{\ln(2)}{10} \approx -0.069$$

14 → 23

Updated model

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Updated model:

$$30e^{-0.069t}$$

How long until I have 1 gram?
 (which t?)

$$1 = 30e^{-0.069t}$$

$$\frac{1}{30} = e^{-0.069t}$$

$$-\ln(30) = -0.069t$$

$$t = \frac{-\ln(30)}{-0.069} \approx 49.29 \text{ yr}$$