"continuous" growth/decay ~ $\underbrace{y_0}_{\text{initial amount}} e^{rt}$ ← growth/decay rate

$$\left(\begin{array}{l} r>0 \sim \text{growth} \\ r<0 \sim \text{decay} \end{array}\right)$$

"percent growth/decay" ~ $\underbrace{y_0}_{\text{initial amount}}(1 \pm \underbrace{p}_{\text{percent growth/decay}})^{\frac{t}{T}}$ ← time period

(⊕ growth, ⊖ decay)

Ex: A ^starting pop. of 300 fish ^are in a lake. Their pop decreases by $\frac{1}{3}$ every 5 years.

Express decay of this population in form

$$a b^{\frac{-t}{T}}$$

$$= \frac{a}{b^{t/T}}$$

33.3% ⇝ $p = 0.333$

decrease ⇝ ⊖

$1 - 0.333 = 0.667$

Recall

$5^{-1} = \frac{1}{5}$

$$\frac{300}{(0.667)^{t/5}} = 300\left(\underbrace{\frac{1}{0.667}}_{3}\right)^{-t/5}$$

Ex: Currently 13 frogs in pond

Frog pop grows expon'tly, doubles every 3 days,

How long (in days) does it take for there to be 200 frogs in pond?

2000 frogs in pond?

$\% = \frac{1}{100}$

$100\% = \frac{100}{100} = 1$

Soln: $y_0(1+P)^{\frac{t}{T}}$

100% increase every 3 days

$$\boxed{P=1}$$

$$= 13\cdot\left(2^{\frac{t}{3}}\right)$$

notice: if $t=3$:

$$13\cdot 2^{\frac{3}{3}} = 13\cdot 2 = 26$$

To get to 200:

$$\underbrace{200}_{\text{target we want to know}} = \underbrace{13\cdot 2^{\frac{t}{3}}}_{\text{model}} \longrightarrow \frac{200}{13} = 2^{\frac{t}{3}}$$

$$\ln\left(\frac{200}{13}\right) = \ln\left(2^{t/3}\right)$$

$$= \frac{t}{3}\ln(2)$$

$$\Rightarrow t = \frac{3\ln\left(\frac{200}{13}\right)}{\ln(2)} \approx \boxed{11.83 \text{ days}}$$

<u>to 2000?</u>

$$2000 = 13 \cdot 2^{\boxed{t/3}}$$

$$\ln\left(\frac{2000}{13}\right) = \frac{t}{3}\ln(2)$$

$$t = \frac{3\ln\left(\frac{2000}{13}\right)}{\ln(2)} \approx 21.80 \text{ days}$$

Ex: A pop. 5.6 million grows at 1.1% per year.

What is the population in $\underbrace{\text{one century}}_{\boxed{100\text{yr}}}$? $\uparrow$ $T=1$

Soln: $1.1\% = \frac{1.1}{100} = 0.011$

Growth formula:

$$\underbrace{5.6(1+0.011)^{\boxed{t} \leftarrow \text{measured in years}}}_{\text{millions of people}} = 5.6(1.011)^{t}$$

pop in 1 century: set $\underline{t=100}$

$$5.6(1.011)^{100} \approx 16.72 \text{ million}$$

Ex: Find doubling time of a city whose pop grows 11% per year

Soln: $P = 11\% = \frac{11}{100} = 0.11$ $T = 1$

$t \sim$ years

Growth model

$$y_0(1+0.11)^t = y_0(1.11)^t$$

Doubling time

$$2\cancel{y_0} = \cancel{y_0}(1.11)^t$$

$$2 = (1.11)^t$$

$$\ln(2) = t\ln(1.11)$$

$$t = \frac{\ln(2)}{\ln(1.11)} \approx 6.64 \text{ years}$$

Ex: Half-life of a radioactive substance is 10 yr.

If you have 30 grams of the substance then how long will it take to get only 1 gram?

Soln: Model as continuous decay:

$$\underbrace{y_0 e^{rt}}_{\text{grams of subst}} \leftarrow \text{years} \longrightarrow 30e^{rt} \leftarrow \boxed{\text{find } r}$$

B/c half life is 10 yr:

$$15 = 30e^{10r}$$

(I have 15 grams at 10 year mark (half-life))

$$\frac{1}{2} = \frac{15}{30} = e^{10r} \qquad \ln\left(\frac{1}{2}\right) = \cancel{\ln e}^{10r})$$

$$\ln\left(\frac{1}{2}\right) = \ln(1) - \ln(2), \quad \ln(1) \to 0$$

$$-\ln(2) = 10r$$

$$r = -\frac{\ln(2)}{10} \approx -0.069^{74\ 718}$$

Updated model

Updated model:

$$30e^{-0.069t}$$

How $\underbrace{\text{long}}_{\text{(which } t\text{?)}}$ until I have 1 gram?

$$1 = 30e^{-0.069t}$$

$$\frac{1}{30} = e^{-0.069t}$$

$$-\ln(30) = -0.069t$$

$$t = \frac{-\ln(30)}{-0.069} \approx 49.29 \text{ yr}$$